

How to Build Mersenne Primes

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Abstract - We use Euclid's formula for PERFECT NUMBERS to develop MERSENNE PRIMES in a very simple way amenable to computers.

A number N is perfect if the sum of the factors of N including 1 and N IS EQUAL TO $2N$. EUCLID has given a formula for PERFECT NUMBERS as $PN = 2^{(N-1)}\{2^N - 1\}$ where $(2^N - 1)$ is a prime.

Incidentally $(2^N - 1)$ is given the name MERSENNE NUMBER and if it works out to be a PRIME then N should necessarily be a prime and is given the name MERSENNE PRIME. We consider prime values only for N chosen from list generated using MUTHKUR's THEOREM which states that (taken from my Ph.D thesis)

THEOREM : If there is a higher MERSENNE prime then that corresponding prime exponent is the sum of lower MERSENNE PRIME EXPONENTS including 2 which is also a MPE.

Proof: Let $Mp1, Mp2, Mp3, \dots, Mp_k$ be a set of Mersenne Primes.

$$\sigma(Mp1.Mp2.Mp3.\dots.Mp_k) = \sigma(Mp1).\sigma(Mp2).\sigma(Mp3).\dots.\sigma(Mp_k)$$

$$= 2^{p1} . 2^{p2} . 2^{p3} \dots 2^{pk} = 2^{(p1+p2+p3+\dots+pk)} - 1 + 1$$

$$= \sigma(MP) \text{ where } P=p1+p2+p3+\dots+pk \text{ is also a prime.}$$

Hence the Theorem.

The formula

$$PN = 2^{(N-1)}\{2^N - 1\} = 2^{(2N-1)} - 2^{(N-1)}$$

and this involves only powers of 2 which a computer can handle very easily.

$$\text{LET } N=2 \quad PN = 8-2 = 6 \text{ which is perfect.}$$

$$\text{LET } N=3 \quad PN = 32 - 24 = 28 \text{ which is perfect.}$$

$$\text{LET } N=5 \quad PN = 2^9 - 2^4 = 512 - 16 = 496$$

Keep on dividing 496 by 2 successively till we reach an odd number, 31 in this case, and add 1 to it and again continue division by 2 successively. If the process reaches 2 and finally 1 then we arrive at a MERSENNE PRIME. Otherwise NOT. Here 5 is a Mersenne prime exponent and 31 is a Mersenne prime. By this, the cumbersome method of testing an odd number with lakhs of digits in it for primality gets reduced to division by 2 successively in a most simple way with definite conclusion whether it is mersenne prime or not.

$$\text{Let } N=7 \quad PN = 2^{13} - 2^6 = 8192 - 64 = 8128 \text{ which is PERFECT.}$$

$$(1+2+4+8+16+32+64+127+254+508+1016+2032+4064+8128 = 16256)$$

Hence 7 is a Mersenne Prime Exponent and 127 is a Mersenne Prime.

Let $N=11$ $PN = 2^{21} - 2^{10} = 2097152 - 1024 = 2096028 = 4 \cdot 52223$

Hence 11 is not a Mersenne Prime Exponent.

Let $N=13$ $PN = 2^{25} - 2^{12} = 33554432 - 4096 = 33550336 = 2^{12} \cdot 8191$

and 8191 is a prime. Hence 13 is a Mersenne Prime Exponent.

Let $N=17$ $PN = 2^{33} - 2^{16} = 8587869056 = 65536 \cdot 131071$

and 131071 is a prime. Hence 17 is a Mersenne Prime Exponent.

Let $N=19$ $PN = 2^{37} - 2^{18} = 137438691328 = 262144 \cdot 524287$

and 524287 is a prime. Hence 19 is a Mersenne Prime Exponent.

This process continues. Finding next higher Mersenne Prime Exponent is half way through for people with super computer facilities and the like.

And in conclusion, I request "The Electronic Frontier Foundation of America "and " GIMPS " to provide me a Scholarship and a Prize money for this theoretical investigation to pursue my research further in this field.

I am a senior citizen 88 years aspiring for lifetime achievement award having

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