# A Hybrid Particle Filter Resampling for a Special Study of Tracking

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**Abstract- This paper proposes a combined method based on Maximum Relative Entropy (MrE) for Particle Filtering (PF) resampling to build system model knowledge in the form of equality constraints. Firstly, MrE approachallows theparticle cloud to be updated by both observed data and constraint moments in a single time step, and impose particular forms on the posterior. To speed up the process of finding the desired distribution for each time step, we derive and find the relationship between Sampling Importance Resampling (SIR) PF and MrE PF by a new term $\frac{e^{\beta f(x)}}{Z}$, called additional equality constraint. The numerical method to determine $\beta$ is introduced by minimized the loss function. This is the same way to applied by incorporating Kullback-Leibler Distance (KLD) resamplingPF and MrE PF. Our experimental based on target tracking show that this methodobtain about 57% increase in performanceof average error. This is also a remarkable result in tracking using PFs.**

**Keywords – Particle filtering, Maximum Relative Entropy, KLD resampling**

## I. Introduction

The focus on the target tracking problem given external momentary knowledge of the system described as constraints. Target tracking has been historically explored due to its application in numerous domains, e.g. plane tracking in military or indoor localization for robots. Traditionally, the problem here is how to pinpoint the target position from the given a sequence of observed data obtained through sensors reliably Due to the nature of white noise perturbations in the environment and internal system of sensors, it is difficult to yield an certain answer except relying on statistical inference to approximate the result. However, that of building the movement model of the target and solving analytic equations for the optimal answer is complicated,Particle Filtering (PF) has been introduced to give the feasible solutions.

To extend and make the target tracking more applicable, some obvious constraints are suggested to be considered on the model of the moving target. one of example of the constraint is the moving of train on a railway. If we know the map of the railway, it is reasonable to express our belief in the form of a probability distribution stretched along the railway, focusing on the direction the moving of trainrather than the sides. This example will be discussed more details in the simulation session.

The solution proposed in this paper will consider only equality constraints inwhich the functional form of the expected value of a function is a constant.

There are several studies on the problem of tracking in the presence of constraints. Firstly, Das et. al [1] introduced the use of optimal transport theory to solve Bayesian filtering issue of nonlinear equality constrained state estimation. The idea is finding transport map between the prior and posterior PDF on measure space, that are optimal regarding cost function specified as the sum of Euclidean distances between samples from these distributions.Secondly, Giffin et. al [2] proposed a series of Kalman filter with MrE to derived analytic solution such as Kalman filter, extended Kalman filter, and the unscented Kalman filter.Thirdly, Xiong et.al [3] proposed constrained PF based on KLD by sampling size test and truncating region with two particles handling strategies for the non-linear dynamic problems. Amor et.al [4]introduced rejection and projection approaches. Rejection method focused on retaining particles that fell within the constrained interval and rejecting all violating constraint region.

While, projection one is applied to impose the particle to be within the constraining interval, the obtained particles are no longer considered as representative samples of the posterior distribution of the state. Finally, our recent research in [5,11] successfully solved the minimum of average number of particles used for KLD PF by finding bound error with Support Vector Machine (SVM) algorithm in [11]. We deployed an architectural model to collect and store error value of each iteration in database, which is used for online training phase.

In this paper, a framework is provided to incorporate MrE to the PF algorithm. This contribution launches a new method to express constraints on posterior which yields better accuracy than that of traditional methods such as Sampling Importance Resampling (SIR) PF [6,9] and KLD PF [3,7,8]. Furthermore, by giving MrE, a new application, we have been filled the gap between theory and practice byproviding a compelling background for testing new ideas of the method

System Model, Proposed MrE PF, Experimental and Conclusion are contents presented in this paper, respectively.

## II. SYSTEMMODEL

Consider the target tracking mathematically in discrete time in [3, 5, 8]which are shown as follows

$$x_k = f_k(x_{k-1}, w_k \tag{1}$$

$$z_k = h_k(x_k, \tag{2}$$

where $f(\cdot)$ and $h(\cdot)$ are nonlinear functions; $x_k$, $z_k$, $w_k$, and $v_k$are state, measurements, process noise, and measurement noise, respectively. The system state,measurement noises and initial state $x_0$ obey non-Gauss distributions

To test the accuracy of the proposed method, we implement a simulation to compare with the SIR PF[6] and KLD PF [3, 7]. The simulation requires tracking a train going on a known 2D railway. Formula for the curve going through the middle of the track is simplified in the form of.

(3)

where $a$ and $b$ are the real number.

## III. PROPOSED ALGORITHM

### *3.1Maximum relative entropy (MrE)*

The principle of maximum Entropy states that the probability distribution which has the highest chance to emerge in the context of prior data is the one with largest entropy. This has been proven mathematically and results in Maximum Entropy method (MaxEnt) in [10], allowing one to generate a satisfied distribution from the uniform background measure given a set of equality constraints. Maximum relative entropy (MrE, also known as ME) is a generalized form of MaxEnt, extending the form of the prior to arbitrary [2]. Furthermore, it has been realized that MrE also produces every aspect of orthodox Bayesian inference, thus becoming the universal updating method to corporate both observed data and moments (constraints) to the distribution.

### *3.2 MrE- PF*

In this part, an argument isintroduced to increaseMrE as an universal updated method. For detail discussion, as in[10], MrE combines the necessary information of the prior for the update process into a single joint distribution. For the PF case, the joint distribution is defined

(4)

where new observations will be put as constraints on $z$ (Bayesian update on data) and moments will be put as constraints on $x$ (MaxEnt update on functional constraints of state distribution). In q.(4), $P$ is denoted *old* and *new* with respect to before and after the update process, respectively

Kolmogorov second axiom for probability requires the joint distribution satisfy the following condition

(5)

Observations will be represented as constraints on the family of posterior. The family of posteriors *P(z,x)* that reflects the fact that *z* is now known to be *z′* is such that

(6)

where is the Dirac delta function. This amounts to an infinite number of constraints: there is one constraint $\lambda$ on *P (z, x)* for each value of the variable *z*.

Moments will be represented as expected value of a function *f* 's equality to a constant *F as*

(7)

In a single time step, for simultaneous update under all constraints, the followed variation form with Lagrange multipliers must be satisfied

$$\delta\{S + \alpha[\int P(z,x)\,dzdx - 1] + \beta[\int P(z,x) f(x)\,dzdx - F] + [\int \lambda(z)\,dz \int P(z,x)\,dx - \delta(z - z') \tag{8}$$

which yields the posterior as

$$P_{new}(x) = P_{old}(x|z') \tag{9}$$

where is determined by

(10)

and

(11)

Comparing to SIR PF in Eq.(9), $P_{new}(x)$and $P_{old}(x)$ are the posterior and prior, respectively; and $\frac{P_{old}(z}{P_{old}|}$is the support *z′* provides for *x′*, and there is a new term accounting for additional equality constraint $\frac{e^{\beta}}{}$. It is important to note that, MrE for PF has its single moment apply sequentially for each time step.

In discrete representation of PF, Eq.(11) becomes

(12)

*3.3 Numerical method to determine* $\boldsymbol{\beta}$

Calculating directly from the implicit eq.(10) is difficult, however we can implement a simple gradient descent algorithm to find the approximating solution in reasonable time. The problem is of solving (11) is the same is minimizing the loss function as

(13)

where is Euclidean norm.Noted that, since PF represents the posterior in discrete form, *L* tends to not give exact *0*. However through experiment, we are usually able to find value such that $\frac{\partial L}{\partial \beta}$, giving closest optimum to*0*.

Calculating derivative of a function *f* is done through numerical differentiation method

(14)

Value will be determined through the iteration formula with step size

(15)

*3.4 Kullback–Leibler Distance (KLD) resampling*

To determine the sampling size through KLD will be introduced [3, 8]. The KLD is to measure the difference between two probability distributions *p* and *q* as

(16)

The maximum likelihood estimation *p* can be specified $\hat{p} = N$, and logarithm of the likelihood ratio is given by

, (17)

where $X_i =$ then we have

(18)

Let*p*( $K(\hat{p}, p)$) means the probability that the KLD between the true distribution and the maximum likelihood estimation based on samples is less or equal to . The derivation eq.(18), we have

*p*( =*p*(2log )=*p*( ) (19)

Let us define $N_{re}$ is the required number of samples can be determined as $N_{re} = X^2_{l-1,1-\delta}$ and the mean particle used criterion is collected as follows

(20)

where $z$ is the upper quantile at (1- ) of the standard normal distribution.

## IV. EXPERIMENTS AND RESULTS

The eq.(3) is used to simulate the train setting up parameters fortracked through 20 steps, with each time step Δk = 1. In a span of a time step, the train can move with hidden velocity $v_k$, results in a stochastic coordinate state $x_k = (c_k^0$,where x-axis component $c_k^0 \sim N(c_{k-1}^0 + v_k \Delta k$ and y-axis component $c_k^1 \sim N(T(c_k^0)$.

In this simulation,values a and b are assigned to 0.25 and 1.5, respectively. Velocity is setto 1 for all *k* *k*; $v_k = 1$ and standard deviation for the transition normaldistributions is set to σ = 0.1. There will be 4 sensors with the error rate ofe ~ N (0, 0.1). The number of particles is initially set to 800.

Table 1. Generate the covariance matrix *Cov*

| |
|---|
| **Algorithm 1:** Generate covariance matrix Cov. Calculate rate of change *r* through numerical method, then determine angle and transformation components U, |
| **Input:** $m_k$<br>-Calculate $r = \partial T$<br>-Calculate $\theta = \arctan$<br>-Calculate $U = \begin{bmatrix} cos\theta & -s \\ sin\theta & cc \end{bmatrix}$<br>-Calculate $\Lambda = |$<br>-Calculate $Cov = U^T$<br>**Output:***Cov* |

The moment constraint on the posterior of each time step is defined as

$$(f(x)) = \sum_x MultivariateN(m_k, Cov).pdf(x) \quad (21)$$

where the state estimate of the particles after transition $m_k$is calculated as

$$m_k = \sum w \tag{22}$$

The covariance matrix Cov is presented in Table 1.

The moment constraint forces the posterior to take the form of a multivariate normal distribution, with its mean at the estimated state after transition and covariance matrix spreading along the gradient vector of Eq.(3).The simulation is conducted through *4* iterations of *500* runs which results in total *2000* simulations for each algorithm. Error of each time step is the Euclidean distance between the estimated state and the train's true position. The average result at each time step is shown in the chart of Figure 1.

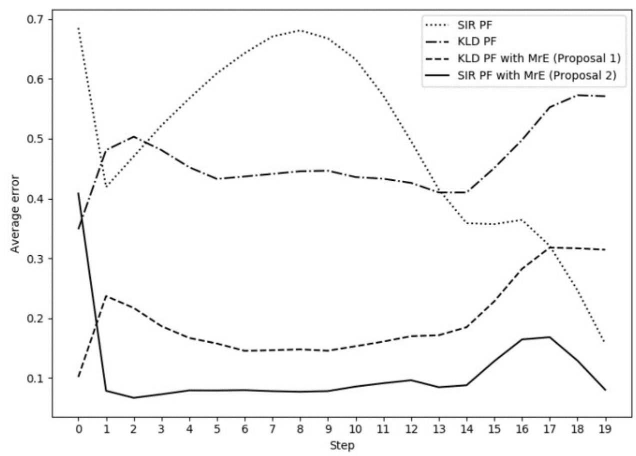


Figure 1. The performance of the average error vs step for all methods

Figure 1 shows the performance of the average error vs. step for SIR PF, KLD PF without MrE and our suggestions such as SIR PF with MrEand KLD PF with MrE. In general, our proposed ideas show an improvement in average error. SIR PF and KLD PF with MrE both achieve a low error and confirmed by the simulations. For example, the gap of average error of KLD PF with MrE and without MrE (KLD PF) is about 0.2m for the whole step. Moreover, the gap of average error of SIR PF with MrE and SIR PF without MrE is about 0.4 m (at step 1) to 0.6 m (at step 9).Numerical comparison of four algorithms is in the Table 2.

Table -2 Average error and runtime for all methods

| | **Average error [m]** | **Runtime [ms]** |
|---|---|---|
| SIR PF [6,9] | 0.493 | 62.4 |
| KLD PF[3,7,8] | 0.461 | 57.6 |
| SIR PF with MrE (Proposal 2) | 0.111 | 72.5 |
| KLD PF with MrE (Proposal 1) | 0.198 | 61.9 |

Table 2verifies that SIR PF with MrE dominates about the performance of the accuracy location (the smallest average error). However, the runtime of this technique needs about 10*ms* more comparing to that ofSIR PF. This time is used to find $\beta$ in MrE algorithm. Furthermore, while KLDPF with MrE has higher error than that of SIR PF with MrE, it proves to has smaller runtime. It obtains about 57% increase in performance. As a result, KLD is a feature allows us to leverage between accuracy and runtime.

## IV.CONCLUSION

In this paper, we propose the MrE for SIR PF and KLD PF methods to improve the accuracy location when compared to traditional methods for tracking problem. Our algorithm is built on the combinations of Maximum relative entropy (MrE), MrE – PF, numerical method to determine β for KLD or SIR resampling. The experiment is run by 4 sensors with the error rate of e ~ N (0, 0.1) and 800 number of particles at the initialization step. The algorithms give improved results on average error deceases from 0.493 to 0.111 for SIR PFwithout and with MrE, respectively.Our future work, this technique will be extended (e.g. implicit transition model as constraints) and applied to our architecture diagram in wireless sensor network.

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