Ml Estimation of Frquecy Failure Indices for Two-Component System in Presence of Ccfs Follows Weibull Law

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Abstract- This study intended to assess the maximum likelihood estimation method for the two component repairable system . The system is assumed to be under the influence of common-cause failures (CCFs). The CCFs and individual failures follows weibull law with occurrence of chance. Numerical evidences are provided to justify the use of M L estimation procedure in the cause of system Frequency Failure functions.
Keyword: Availability, series, parallel system, CCFS failure, MLE.

I. INTRODUCTION

System Reliability plays a vital role in nuclear power plants, electrical, electronics and industrial sectors . the common cause failures have identified in recent time (1971) which have a significant contribution to highly risk in complexity systems. like nuclear power plants. The multi-component failures due to external causes like radiation, humidity etc this type causes is called CCF . CCFs are greatly reducing the reliability indices under its influence. Billiton &Allan [1983] discussed the role of CCFS used BFR model for CCFS in the area of nuclear power plants. The Quantification and estimation of CCFs rates were discussed by A.A.Chari[1991]. The number of failures in relative time of exposure of each component are used in Various papers of James-Stein (1992). U S Manyam & A.A.Chari [2008] have studied the concept of CCFs to arrive at exponential law . The expression of Frequency Failure F(t) functions using markovian approach. This paper attempts the estimation of Frequency Failure F(t) functions for parallel and series system in the context of Common Cause Failures to arrive at weibul-law.

II. ASSUMPTIONS

1. The system has two components, which are stochastically independent.
2. The system is affected by individual as well as common cause failures.
3. The components in the system will fail singly at the constant rate βa and failure probability is P1
4. The components may fail due to common causes at the constant rate βc and with failure probability is P2 such that P1 +P2 = 1.
5. Time occurrences of CCS failures and individual failures follow Wei-bull law.
6. The individual failures and CCS failures occurring independent of each other.
7. The failed components are serviced singly and service time follows exponential distribution with rate of service .

III. NOTATIONS

βI : Individual failure rate.

βc : Common cause failure rate.

μ:Service rate of individual components

Fs(t) : Steady state Frequency of Failure function of series system.

$\hat{F}$s(t): ML Estimate of steady state Frequency Failure function of series system.

Fp (t): Steady state Frequency Failure function for parallel system.

$\hat{F}$p(t): ML Estimate of steady state frequency of Failure function for parallel system.

$\hat{\beta}_I$:sample estimation of individual failure rate

$\hat{\beta}_c$:sample estimation of common cause failure rate

$\hat{\mu}$: Sample estimate of service time of the components

n = Sample size.
N = Number of simulated samples

IV. MODEL

The assumptions of Markova model can be to drive and be formulated the Reliability function R(t) under the influence of individual as well as CCF. The quantities β_0, β_1, β_2 are as follows

$\beta_0 = \beta_I P_1$, $\beta_1 = 2\beta_I P_1$ $\beta_2 = \beta_c P_2$, $\mu_1 = \mu$, $\mu_2 = 2\mu$,

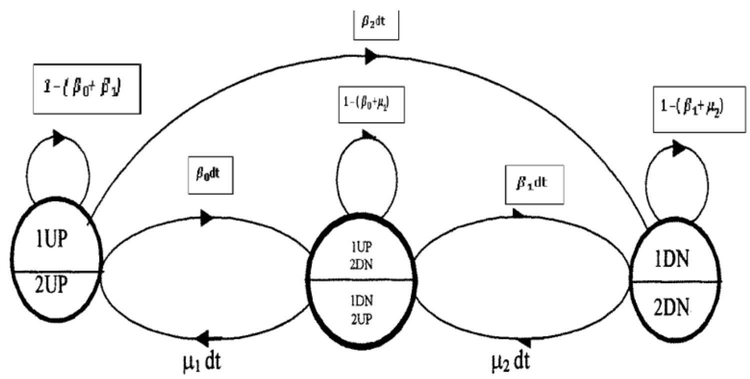

Fig. 3.1: Markov Graph For Two Component System With Individual and Common Cause Failures.

From the Markov graph the equations were formed and the probabilities of the Various state of the systems i.e. Po(t), P1 (t), P2(t) are derived (Chari [1991]).

4.1 Two Component Identical System Frequency Of Failures

The frequency of failures function for series and parallel systems are derive using the probabilities mentioned in the section

(a) Series System

Thus, the expression of frequency of failure function for series system is give by

$$FF_s(t) = \frac{\left(2\mu_0^2 \quad (2\beta_1+\beta_2)\right)}{\left(\mu_1(\mu_0+\beta_0+\beta_2)+ \beta_2+\beta_0(\beta_1+\beta_2)\right)} \tag{1}$$

Where

$\beta_0 = \beta_I P_1$, $\beta_1 = 2\beta_I P_1$ $\beta_2 = \beta_c P_2$ $\mu_0 = \mu$, $\mu_1 = 2\mu$,

β_0, β_1, β_2,μ_0 μ_1 P1&P2 are individual failures rate, Common cause failure rate, repair rate and probability of occurrence.

(b) Parallel System

The expression of Frequency of failure function of parallel system is

$$FF_p(t) = \frac{\left(\mu_1(2\beta_2(\mu_0+\beta_0)+\beta_0^2)\right)}{\left(\mu_1(\mu_0+\beta_1+\beta_2)+\mu_0\,\beta_2+\beta_0(\beta_1+\beta_2)\right)} \tag{2}$$

Where

$\beta_0 = \beta_I P_1$, $\beta_1 = 2\beta_I P_1$ $\beta_2 = \beta_c P_2$ $\mu_0 = \mu$, $\mu_1 = 2\mu$,

β_0, β_1, β_2,μ_0 μ_1 P1&P2 are individual failures rate, Common cause failure rate, repair rate and probability of occurrence.

V. ESTIMATION OF RELIABILITY FUNCTION-ML ESTIMATION APPROACH

This section discusses the Maximum likelihood estimation approach for estimating Reliability function of two component parallel and series systems, which is under the influence of Individual as well as common cause failures.
Let X1, X2, X3...Xn,, be a sample of ‘n' number of times between individual failures which will obey weibul law.
Let Y1 Y2, Y3 ...Yn,, be a sample of ‘n number of times between common cause system failures assume to follow Weibul law.

Let Z1 Z2, Z3 ...Zn,, be a sample of 'n` number of times between service of the component assume to follows exponential law.

$$\frac{\sum_{i=1}^{n} x^k \ln x_i}{\sum_{i=1}^{n} x^k} - \frac{1}{k} - \frac{\sum_{i=1}^{n} \ln x}{n} = 0 \qquad \frac{\sum_{i=1}^{n} y^k \ln y_i}{\sum_{i=1}^{n} y^k} - \frac{1}{k} - \frac{\sum_{i=1}^{n} \ln yi}{n} = 0$$

$$\hat{\mu} = \frac{1}{\frac{\sum z_i}{n}} \quad \hat{\beta}_I = \frac{\sum_{i=1}^{n} x_i^K}{n} \quad \hat{\beta}_c = \frac{\sum_{i=1}^{n} y_i^K}{n} \tag{3}$$

are sample estimates of rate of the individual failure rate ($\hat{\beta}_I$) ,common cause failure rate ($\hat{\beta}_c$) and service rate $\hat{\mu}$ of the components respectively. under precedence of individual as well as CCS failures.

VI. ESTIMATION OF TWO COMPONENT SYSTEM FREQUENCY FAILURES M L ESTIMATION APPROACH

The frequency failure functions for two component identical series and parallel systems by using ML approach as follows.

(a) Series System

Thus, the expression of frequency of failure function for series system is give by

$$\widehat{FF}_s(t) = \frac{\left(2\widehat{\mu_0}^2(\hat{\beta}_1+\hat{\beta}_2)\right)}{\left(\widehat{\mu_1}(\widehat{\mu_0}+\hat{\beta}_1+\hat{\beta}_2)+\widehat{\mu_0}\,\hat{\beta}_2+\hat{\beta}_2(\hat{\beta}_1+\hat{\beta}_2)\right)} \tag{4}$$

Where

$\hat{\beta}_0 = \hat{\beta}_I P_1$, $\hat{\beta}_1 = 2\hat{\beta}_I P_1$ $\hat{\beta}_2 = \hat{\beta}_c P_2$ $\hat{\mu}_0 = \mu$, $\hat{\mu}_1 = 2\hat{\mu}$,

are sample estimates of rate of the individual failure rate ($\hat{\beta}_I$) ,common cause failure rate ($\hat{\beta}_c$) and service rate $\hat{\mu}$ of the components respectively. under precedence of individual as well as CCS failures.

(b) Parallel System:

Thus, the expression of frequency of failure function for parallel system is give by

$$\widehat{FF}_p(t) = \frac{\left(2\widehat{\mu_0}(2\hat{\beta}_2(\widehat{\mu_0}+\hat{\beta}_1)+\hat{\beta}_0^2)\right)}{\left(\widehat{\mu_1}(\widehat{\mu_0}+\hat{\beta}_1+\hat{\beta}_2)+\widehat{\mu_0}\hat{\beta}_2 \;+\hat{\beta}_0(2\hat{\beta}_1+\hat{\beta}_2)\right)} \tag{5}$$

Where

$\hat{\beta}_0 = \hat{\beta}_I P_1$, $\hat{\beta}_1 = 2\hat{\beta}_I P_1$ $\hat{\beta}_2 = \hat{\beta}_c P_2$ $\hat{\mu}_0 = \mu$, $\hat{\mu}_1 = 2\hat{\mu}$,

are sample estimates of rate of the individual failure rate ($\hat{\beta}_I$) ,common cause failure rate ($\hat{\beta}_c$) and service rate $\hat{\mu}$ of the components respectively. under precedence of individual as well as CCS failures.

VII. SIMULATION AND VALIDITY :

For a range of specified values of the rates of individual (βI), common cause failures(βc) and service rates(μ) and for the samples of sizes n = 5 (5) 30 are simulated using computer package developed in this paper and the sample estimates are computed for N = 10000 (10000) 100000 and mean square error (MSE) of the estimates for Fs(t) , FP(t) , were obtained and given in tables [Tab.1 Tab.2,] The tables and graphs are seen in the. For large samples Maximum Likelihood estimators are undisputedly better since they are CAN estimators. However it is interest to note that for a sample size as low as five (n=5) also M L estimate is still seen to be reasonably good giving near accurate estimate in this case. This shows that ML method of estimator is quite useful in this context.

Table-1:Results of the simulations for frequency failure function for series system with
$\beta_1 = 0.02$ $\beta_c = 0.03$ $\mu = 1$ p1= 0.02 k =1

Sample size = 5					Sample size = 10				
N	Fs(t)	$\hat{F}_s(t)$	s.s	M.S.E	N	Fs(t)	$\hat{F}_s(t)$	s.s	M.S.E
10000	0.028902	0.027284	0	0.000016	10000	0.028902	0.023022	0	0.000059
20000	0.028902	0.030188	0	0.000009	20000	0.028902	0.012754	0	0.000114
30000	0.028902	0.024643	0	0.000025	30000	0.028902	0.028774	0	0.000001
40000	0.028902	0.029497	0	0.000003	40000	0.028902	0.028718	0	0.000001
50000	0.028902	0.043647	0	0.000066	50000	0.028902	0.034572	0	0.000025
60000	0.028902	0.032276	0	0.000014	60000	0.028902	0.013551	0	0.000063
70000	0.028902	0.042812	0	0.000053	70000	0.028902	0.018179	0	0.000041
80000	0.028902	0.022889	0	0.000021	80000	0.028902	0.030463	0	0.000006
90000	0.028902	0.025626	0	0.000011	90000	0.028902	0.020321	0	0.000029
100000	0.028902	0.006475	0	0.000071	100000	0.028902	0.025212	0	0.000012

Sample size = 15					Sample size = 20				
N	Fs(t)	$\hat{F}_s(t)$	s.s	M.S.E	N	Fs(t)	$\hat{F}_s(t)$	s.s	M.S.E
10000	0.028902	0.033993	0	0.000051	10000	0.028902	0.027949	0	0.00001
20000	0.028902	0.024078	0	0.000034	20000	0.028902	0.025696	0	0.000023
30000	0.028902	0.018784	0	0.000058	30000	0.028902	0.02389	0	0.000029
40000	0.028902	0.029519	0	0.000003	40000	0.028902	0.025281	0	0.000018
50000	0.028902	0.030023	0	0.000005	50000	0.028902	0.021338	0	0.000034
60000	0.028902	0.029361	0	0.000002	60000	0.028902	0.025425	0	0.000014
70000	0.028902	0.024992	0	0.000015	70000	0.028902	0.027152	0	0.000007
80000	0.028902	0.018935	0	0.000035	80000	0.028902	0.033102	0	0.000015
90000	0.028902	0.031646	0	0.000009	90000	0.028902	0.024866	0	0.000013
100000	0.028902	0.028686	0	0.000001	100000	0.028902	0.033512	0	0.000015

Sample size = 25					Sample size = 30				
N	Fs(t)	$\hat{F}_s(t)$	s.s	M.S.E	N	Fs(t)	$\hat{F}_s(t)$	s.s	M.S.E
10000	0.028902	0.025301	0	0.000036	10000	0.028902	0.022149	0	0.000068
20000	0.028902	0.027603	0	0.000009	20000	0.028902	0.029838	0	0.000007
30000	0.028902	0.032151	0	0.000019	30000	0.028902	0.022457	0	0.000037
40000	0.028902	0.025787	0	0.000016	40000	0.028902	0.027138	0	0.000009
50000	0.028902	0.023735	0	0.000023	50000	0.028902	0.031428	0	0.000011
60000	0.028902	0.023382	0	0.000023	60000	0.028902	0.029578	0	0.000003
70000	0.028902	0.02554	0	0.000013	70000	0.028902	0.028299	0	0.000002
80000	0.028902	0.028001	0	0.000003	80000	0.028902	0.020943	0	0.000028
90000	0.028902	0.028701	0	0.000001	90000	0.028902	0.024161	0	0.000016
100000	0.028902	0.028793	0	0	100000	0.028902	0.027209	0	0.000005

Table-2:Results of the simulations for frequency failure function of parallel system with
$\beta_1 = 0.1$ $\beta_c = 0.2$ $\mu = 1$ P1= 0 k =1

Sample size = 5					Sample size = 10				
N	FP(t)	$\hat{F}_p(t)$	s.s	M.S.E	N	FP(t)	$\hat{F}_p(t)$	s.s	M.S.E
10000	0.413223	0.348313	0	0.000649	10000	0.413223	0.365598	0	0.000476
20000	0.413223	0.380439	0	0.000232	20000	0.413223	0.395144	0	0.000128
30000	0.413223	0.401052	0	0.00007	30000	0.413223	0.333294	0	0.000461
40000	0.413223	0.364046	0	0.000246	40000	0.413223	0.352707	0	0.000303
50000	0.413223	0.238645	0.000001	0.000781	50000	0.413223	0.330773	0	0.000369
60000	0.413223	0.358694	0	0.000223	60000	0.413223	0.417039	0	0.000016
70000	0.413223	0.312333	0	0.000381	70000	0.413223	0.419099	0	0.000022
80000	0.413223	0.35455	0	0.000207	80000	0.413223	0.331292	0	0.00029
90000	0.413223	0.380097	0	0.00011	90000	0.413223	0.374636	0	0.000129
100000	0.413223	0.440861	0	0.000087	100000	0.413223	0.371795	0	0.000131

Sample size = 15					Sample size = 20				
N	FP(t)	$\hat{F}_p(t)$	s.s	M.S.E	N	FP(t)	$\hat{F}_p(t)$	s.s	M.S.E
10000	0.413223	0.332251	0.000001	0.00081	10000	0.413223	0.356067	0	0.000572
20000	0.413223	0.357257	0	0.000396	20000	0.413223	0.364325	0	0.000346
30000	0.413223	0.388458	0	0.000143	30000	0.413223	0.36937	0	0.000253
40000	0.413223	0.361807	0	0.000257	40000	0.413223	0.355978	0	0.000286
50000	0.413223	0.315676	0	0.000436	50000	0.413223	0.377867	0	0.000158
60000	0.413223	0.303862	0	0.000446	60000	0.413223	0.347953	0	0.000266
70000	0.413223	0.356513	0	0.000214	70000	0.413223	0.362681	0	0.000191
80000	0.413223	0.378412	0	0.000123	80000	0.413223	0.323227	0	0.000318
90000	0.413223	0.346846	0	0.000221	90000	0.413223	0.333081	0	0.000267
100000	0.413223	0.360682	0	0.000166	100000	0.413223	0.334692	0	0.000248

Sample size = 25					Sample size = 30				
N	FP(t)	$\hat{F}_p(t)$	s.s	M.S.E	N	FP(t)	$\hat{F}_p(t)$	s.s	M.S.E
10000	0.413223	0.334991	0.000001	0.000782	10000	0.413223	0.374152	0	0.000391
20000	0.413223	0.354958	0	0.000412	20000	0.413223	0.317891	0	0.000674
30000	0.413223	0.314356	0	0.000571	30000	0.413223	0.379273	0	0.000196
40000	0.413223	0.344829	0	0.000342	40000	0.413223	0.347896	0	0.000327
50000	0.413223	0.366392	0	0.000209	50000	0.413223	0.32693	0	0.000386
60000	0.413223	0.364102	0	0.000201	60000	0.413223	0.32749	0	0.00035
70000	0.413223	0.333923	0	0.0003	70000	0.413223	0.357899	0	0.000209
80000	0.413223	0.353382	0	0.000212	80000	0.413223	0.378495	0	0.000123
90000	0.413223	0.301725	0	0.000372	90000	0.413223	0.369242	0	0.000147
100000	0.413223	0.330697	0	0.000261	100000	0.413223	0.355965	0	0.000181

VIII. CONCLUSIONS:

This paper attempts to evaluate the estimate of the Availability for transient and steady state function s in the presence of common cause and individual failure. The ML method proposed here is giving almost accuracy estimation in case of sample size 10 and above which is verified by the simulation in the absence analytical approach. Also these results suggested the ML estimate is reasonable good and gives accurate estimates even for sample size n=5 therefore this paper identifies the use of thee an ML method of estimator justified through empirical means estimation of the Availability of two component system in presence of CCFs as well as individual failures.

IX. REFERENCE

[1] Fleming, K. N. [1974]A reliability model for common mode failure in redundant safety systems. General Atomic Report, GA-13284, Pittsburgh, PA.

[2] Steverson, J.A & Atwood, C.L.[1983] Common cause failure rates for values Journal of US Nuclear Regulatory Commission Report NUREGCR-2770.EGG-Ea-5485.

[3] Atwood, C.L.[1984] Approximate tolerance intervals, based on Maximum Likelihood Estimates Journal of the American StatisticalAssociation,. pages 450-465

[4] Atwood, C. L. [1986] The binomial failure rate common cause model. Techno metrics 28, pp. 139-148.

[5] Smith, A.M. et al., [1988] Defensive Strategies for Reducing Susceptibility to Common Cause Failures, EPRI NP-5777, Electric Power Research Institute, PaloAlto, CA

[6] Crowder, M. J., Kimber, A. C., Smith, R. L. and Sweeting, T. J. (1991) Statistical Analysis of Reliability Data. Chapman & Hall, London

[7] G. Y. Sagar, Y. Raghunatha Reddy et al., (2010) AVAILABILITY MEASURES FOR THREE COMPONENT SYSTEM WITH HUMAN ERRORS AND CCS FAILURE, IJCSR International Journal of Computer Science and Research, Vol. 1 Issue 1, 2010

[8] Sagar, G. Y. , Reddy, Y. R. et al (2010), Some reliability measures for three component-system in the presence of CCS failures and human errors, Journal of Interdisciplinary Mathematics, Vol.3 Issue 12, 2010

[9] B. S. Dhillon [1989] Stochastic analysis of a parallel system with commoncause failures and critical human errors, Microelectronics and Reliability,Volume 29,Issue 4,1989, Pages 627637

[10] Balakrishnan, N. Lin, & Ping-Shing Chan [2005] A comparison of two simple prediction intervals for exponential distribution Journal of Reliability, IEEE Transactions on Publication Date: March 2005 Volume: 54, Issue: 1, page(s): 27- 33.

[11] Balakrishnan, N. & Varadan, J. [1991] Approximate MLEs for the location and scale parameters of the extreme value distribution with censoring. Journal of Reliability, IEEE Transactions on Publication Date: Jun 1991,Volume: 40, Issue: 2, page(s): 146-151.209

[12] G.Y. Sagar, Y.R. Reddy, C. Umashankar (2011), Some reliability measures for three component system in the presence of CCS failures and human errors Quality control and applied statistics, ISSN 0033-5207, Vol. 56, Nº. 1-2, 2011, págs. 149-150

[13] Basu, A JP [1964] Estimation of reliability for some distribution useful in life testing Technometrics,vol:6,1964, page(s) 215-219.

[14] Bhattacharya,S.K [1969] Bayesian approach to life testing and reliabilityestimation journal of American Statistical Associate,vol:62, 1969,page(s):319-327.

[15] Billinton, R & Allan,R.N [1983] Reliability Evaluation of Engineering Systems; Concepts and Techniques ", Plenum Press