Assessment of Electromagnetic Far-Field radiated above a Two-layer Conducting Earth

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Abstract - The electromagnetic fields (EMFs) transmitted from a circular loop antenna (a vertical magnetic dipole (VMD)) placed on a specific height over multi-layer conducting earth at high frequencies and its image source fields have been represented. The integral representations of image source fields were evaluated by using simple approach. The solution of the linear Maxwell's equations in the frequency domain were gotten, utilizing Hertz vector to decreasing the field integrals to mixes of known Sommerfeld integrals (SIs), which is favorable over the past numerical methods, considering initial, boundary, and transition conditions. The solution determines the EMFs in the conductive media which consists of primary and scattered waves. The outcomes acquired from this paper can be utilized to assess numerical arrangements of various complexities, for example the underlying multilayered media. In addition, the derived formulation would be useful for interpreting airborne EM systems, and will be helpful for remote detecting particularly when the transmitter is near the surface.
Keywords – Electromagnetic Fields (EMFs) Radiation, Far-field, Earth–air electromagnetic (EM) propagation, Multilayered media, Vertical magnetic dipole (VMD), Sommerfeld integral (SI).

I. INTRODUCTION

The electromagnetic fields (EMFs) radiated from loop antenna source placed over or inside multilayered earth structure attracted great attention of numerous researchers since its initial beginning. This is because its importance in wireless engineering and technology, with applications in radio communication systems, radio direction finding devices, and land mobile radio systems, geophysics, and remote sensing [1-7].

The early classical solution of the problem of a circular loop antenna has been treated by Wait [8, 9]. It is well known that the Sommerfeld integrals (SIs) describing the field components cannot be exactly evaluated and their numerical integration is made difficult by the presence of highly oscillatory terms in the integrands when the source and the field point are far separated.

The classical approach in finding the solution was to apply Fourier inversion to the formal time harmonic solution, leading to a representation in the form of double infinite integral. Accurate numerical evaluation of such an expression is difficult and impractical. By utilizing a quasi-static approach, Wait [10] has been change these integrals into asymptotic structures and he obtained closed-form solution for the fields of a loop source located on a two-layered earth which are which are works well for late times, but it is not optimal for computational efficiency. By applying a similar method, Botros and Mahmoud [11] has been gotten straightforward expressions for the transient responses between loops situated on the two-layered earth. The solution was based on comparing the distances encountered with the significant free space wavelengths, and neglect the displacement currents in the air region. Mahmoud et al. [12] formulated the problem of radiation from a VMD on the surface of two-layered ground and a complete time-domain field has been derived analytically. Taking into consideration the displacement current in the air region, but those in the ground have been neglected. Bishay et al. [13] used a small horizontal loop source on a two- layered earth's model and presented a complete frequency-domain wave solution for the fields, where the displacement currents in all regions are accounted for. Despite their analytical expressions were useful and applicable for the near-field region, but the equations cannot be executed effectively because of the presence of poles and branch cuts as well as of these singularities to the saddle point.

Our objective in this paper is to obtain the complete frequency-domain wave solution for the electromagnetic fields radiated from a small horizontal loop antenna (VMD) placed at a height h over a two-layered earth's model, by using simple approach [14]. The computation method takes into consideration the effects of both conduction as well as displacement currents, and is well suitable for any position of the source loop either in the air or on the surface of the model, in contrary to the earlier methods which face convergence problem.

II. GENERAL DESCRIPTION

The earth model and measurement system under consideration are shown in Figure1.

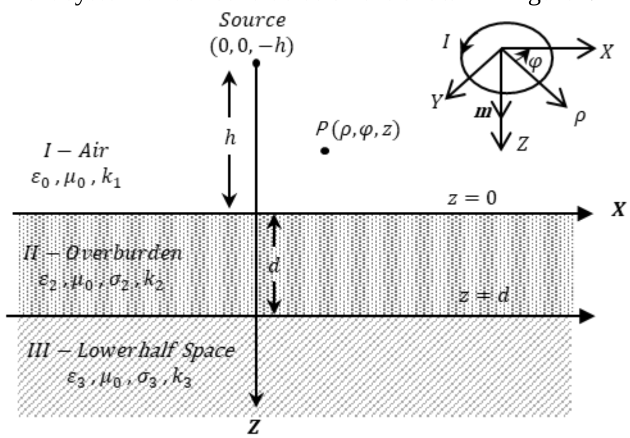

Figure 1: Two-layer earth model and the coordinate system used in the analysis.

The earth's adopted model consists of a homogeneous overburden slab of thickness (d) above the half-space. The coordinate system (x, y, z) with z axis directed vertically downward is taken into consideration. The cylindrical coordinate system (ρ, φ, z) also has been used frequently to solve the equations using circular symmetry. $\varepsilon_i, \mu_i, \sigma_i, k_i$ denote permittivity, permeability, conductivity, and propagation constant, where i = 0, 1, 2 denotes air ($-h \leq z \leq 0$), overburden slab ($0 \leq z \leq d$), and lower half space ($d \leq z \leq \infty$), respectively. The magnetic permeability value is taken to be equal to that of free space throughout. Harmonic factor $e^{i\omega t}$ is implied, and IdA units are used everywhere. The source is a VMD (an infinitesimal horizontal circular loop of radius a, carrying a current I) placed at height $z = -h$ above the surface of layered earth (in the air region: $-\infty \leq z \leq 0$), and the observing point $P(\rho, \varphi, z)$ is located at any distance in the each region.

III. ANALYTICAL FORMULATION

In this section, analytical formulations for the calculation of the electromagnetic fields (EMF) radiating from a VMD located at height "h" in the air region above the surface of layered earth are derived. The geometry of the problem is illustrated in Fig.1. As it has been noted, there are three primary configurations of interest, as a result of different combinations of the locations of the VMD and the observation point. The dipole is assumed to be located on the z-axis. In the type of propagation problems considered here, for analysis restricted to the far field, it can be assumed that the magnitudes of "z" and "d" are much smaller than that of "ρ" (the radial distance between the dipole and the observation point). Also note that primed quantities are associated with the source and suppressed in all the formulations.

3.1 Case I (in the air region $-\infty \leq z \leq 0$):

Consider the simple case when the source and observation point $P(\rho, \varphi, z)$ are both located in the upper layer (air). We take z $\neq 0$, the expressions of the field components, namely $E_{\varphi o}, H_{\rho o}, H_{zo}$ can be written as

$$E_{\varphi_0}(\rho, z) = \frac{-i\omega\mu_0 IdA}{8\pi}\int_{-\infty}^{\infty}\left[\frac{\lambda}{u_0}\, e^{-u_0(z+h)} + f_0(\lambda)\, e^{u_0(z-h)}\right]\lambda\, H_1^{(2)}(\lambda\rho)d\lambda \tag{1a}$$

$$H_{\rho_0}(\rho, z) = \frac{-IdA}{8\pi}\int_{-\infty}^{\infty}\left[\frac{\lambda}{u_0}\, e^{-u_0(z+h)} + f_0(\lambda)\, e^{u_0(z-h)}\right]\lambda\, u_0\, H_1^{(2)}(\lambda\rho)d\lambda \tag{1b}$$

$$H_{z_0}(\rho, z) = \frac{IdA}{8\pi}\int_{-\infty}^{\infty}\left[\frac{\lambda}{u_0}\, e^{-u_0(z+h)} + f_0(\lambda)\, e^{u_0(z-h)}\right]\lambda^2\, H_0^{(2)}(\lambda\rho)d\lambda \tag{1c}$$

where $u_0 = \sqrt{\lambda^2 + k_0^2}$ (the root with positive real part must be taken), $H_0^{(2)}(\lambda\rho)$, $H_1^{(2)}(\lambda\rho)$ are the Zeroth and One order Hankel function of the second kind, and f_1 is constant function which can be arranged and computed easily from the boundary conditions as follows:

$$f_0(\lambda) = \frac{\lambda}{u_0}\left[\frac{r_{01} + r_{12}\, e^{-2u_1 d}}{1 + r_{01}\, r_{12}\, e^{-2u_1 d}}\right] \tag{2}$$

where

$$r_{01}=\frac{u_{01}^{-}}{u_{01}^{+}}=\frac{u_0-u_1}{u_0+u_1},\quad r_{12}=\frac{u_{12}^{-}}{u_{12}^{+}}=\frac{u_1-u_2}{u_1+u_2} \tag{3}$$

r_{01} and r_{12} are Fresnel reflection coefficients at the interfaces z = 0 , d, respectively.
After some manipulations, for example the expression of the magnetic field H_{z_0}, given by (1c) can be written in the following form

$$H_{z_0}(\rho,z)=\frac{IdA}{8\pi}\int_{-\infty}^{\infty}\frac{\lambda}{u_0}e^{-u_0(z+h)}\left[\frac{1-\frac{u_{10}^{-}u_{12}^{-}}{u_{10}^{+}u_{12}^{+}}e^{-2u_1d}-\frac{u_{10}^{-}}{u_{10}^{+}}e^{2u_0z}+\frac{u_{12}^{-}}{u_{12}^{+}}e^{2u_0z-2u_1d}}{1-\frac{u_{10}^{-}u_{12}^{-}}{u_{10}^{+}u_{12}^{+}}e^{-2u_1d}}\right]\lambda^2H_0^{(2)}(\lambda\rho)\,d\lambda \tag{4}$$

Using the relations

$$\left|\frac{u_{10}^{-}u_{12}^{-}}{u_{10}^{+}u_{12}^{+}}e^{-2u_1d}\right|<1 \text{ and } \frac{1}{1-X}=\sum_{m=0}^{\infty}X^m \tag{5}$$

equation (4) can be written as

$$H_{z_0}(\rho,z)=\frac{IdA}{8\pi}\int_{-\infty}^{\infty}\frac{\lambda}{u_0}e^{-u_0(z+h)}\left[1-\frac{u_{10}^{-}u_{12}^{-}}{u_{10}^{+}u_{12}^{+}}e^{-2u_1d}-\frac{u_{10}^{-}}{u_{10}^{+}}e^{2u_0z}+\frac{u_{12}^{-}}{u_{12}^{+}}e^{2u_0z-2u_1d}\right] \times\left[\sum_{m=0}^{\infty}\left(\frac{u_{10}^{-}u_{12}^{-}}{u_{10}^{+}u_{12}^{+}}e^{-2u_1d}\right)^m\right]\lambda^2H_0^{(2)}(\lambda\rho)\,d\lambda \tag{6}$$

Therefore, we may assume that

$$H_{z_0}(\rho,z)=\frac{IdA}{8\pi}\left(\sum_{m=0}^{\infty}[A(m)+B(m)+C(m)+D(m)]\right) \tag{7}$$

where

$$A(m)=\int_{-\infty}^{\infty}\frac{\lambda^3}{u_0}\left(\frac{u_{10}^{-}u_{12}^{-}}{u_{10}^{+}u_{12}^{+}}\right)^m e^{-u_0(z+h)-2u_1md}H_0^{(2)}(\lambda\rho)\,d\lambda \tag{8a}$$

$$B(m)=\int_{-\infty}^{\infty}\frac{-\lambda^3}{u_0}\left(\frac{u_{10}^{-}u_{12}^{-}}{u_{10}^{+}u_{12}^{+}}\right)^{m+1} e^{-u_0(z+h)-2u_1(m+1)d}H_0^{(2)}(\lambda\rho)\,d\lambda \tag{8b}$$

$$C(m)=\int_{-\infty}^{\infty}\frac{-\lambda^3}{u_0}\left(\frac{u_{10}^{-}}{u_{10}^{+}}\right)^{m+1}\left(\frac{u_{12}^{-}}{u_{12}^{+}}\right)^{m} e^{-u_0(-z+h)-2u_1md}H_0^{(2)}(\lambda\rho)\,d\lambda \tag{8c}$$

$$D(m)=\int_{-\infty}^{\infty}\frac{-\lambda^3}{u_0}\left(\frac{u_{10}^{-}}{u_{10}^{+}}\right)^{m}\left(\frac{u_{12}^{-}}{u_{12}^{+}}\right)^{m+1} e^{-u_0(-z+h)-2u_1(m+1)d}H_0^{(2)}(\lambda\rho)\,d\lambda \tag{8d}$$

To calculate the vertical magnetic far–field in air, we using the simple approach [14], when the condition $|k_1|\gg|k_0|$ is satisfied, we have

$$\frac{u_{10}^{-}}{u_{10}^{+}}\approx e^{-u_0a},\qquad \frac{2u_0}{u_{10}^{+}}\approx 1-e^{-u_0a} \tag{9}$$

where $a=\frac{2i}{k_1}$

Using the first formula of (8) and arranging (7a) properly, we get

$$A(m)=\int_{-\infty}^{\infty}\left[\lambda^2\left(\frac{u_{12}^{-}}{u_{12}^{+}}\right)^m e^{-2u_1md}\right]\left[\frac{\lambda}{u_0}e^{-u_0(ma+z+h)}H_0^{(2)}(\lambda\rho)\right]d\lambda \tag{10}$$

When $\rho\to\infty$, there is an approximation formula for the Hankel function [15]

$$H_n^{(2)}(\lambda\rho)\approx\sqrt{\frac{2}{\pi\lambda\rho}}\,e^{-i\left(\lambda\rho-\frac{n\pi}{2}-\frac{\pi}{4}\right)} \tag{11}$$

Also, when $\rho\to\infty$, the first factor of (10) is slowly varying while the second one is rapidly varying.
The location of the stationary phase point λ_{01} of the rapidly varying part is given by

$$\frac{\partial}{\partial\lambda}i\left[-\lambda\rho-(ma+z+h)\sqrt{k_0^2-\lambda^2}\right]\Big|_{\lambda=\lambda_{01}}=0 \tag{12}$$

The solution of (12) is

$$\lambda_{01}=\frac{\rho k_0}{r_1},\quad\text{with}\quad r_1=\sqrt{\rho^2+(ma+z+h)^2} \tag{13}$$

Using the simple approach and considering the following Sommerfeld integral

$$\frac{e^{-ikr}}{r}=\frac{1}{2}\int_{-\infty}^{\infty}\frac{\lambda}{u}e^{-u|z|}H_0^{(2)}(\lambda\rho)\,d\lambda \tag{14}$$

Then, the value of the rapidly varying part in equation (10) is given by

$$\frac{e^{-ik_0 r_1}}{r_1} \tag{15}$$

Most of the contribution to the integral in (10) is from the vicinity of the stationary phase point $\lambda = \lambda_{01}$ if the slowly varying part has no singularities in the vicinity of λ_{01}, then the value of the slowly varying part in equation (10) is given as

$$\frac{k_0^2\,\rho^2}{r_1^2}\left(\frac{1-n}{1+n}\right)^m e^{-2u_1 d} \tag{16}$$

Upon substituting (15) and (16) in (10) we have

$$A(m) = \frac{2k_0^2\,\rho^2}{r_1^3}\left(\frac{1-n}{1+n}\right)^m e^{-ik_0 r_1 - 2ik_1 md} \tag{17}$$

where $n = \frac{k_2}{k_1}$, and the approximation $u_1 \approx i\,k_1$ is used.

By using the same procedure, we can obtain

$$B(m) = \frac{-2k_0^2\,\rho^2}{r_2^3}\left(\frac{1-n}{1+n}\right)^{m+1} e^{-ik_0 r_2 - 2ik_1 (m+1)d} \tag{18a}$$

$$C(m) = \frac{-2k_0^2\,\rho^2}{r_3^3}\left(\frac{1-n}{1+n}\right)^{m} e^{-ik_0 r_3 - 2ik_1 md} \tag{18b}$$

$$D(m) = \frac{2k_0^2\,\rho^2}{r_4^3}\left(\frac{1-n}{1+n}\right)^{m+1} e^{-ik_0 r_4 - 2ik_1 (m+1)d} \tag{18c}$$

where

$r_2 = \sqrt{\rho^2 + ((m+1)a + z + h)^2}$, $r_3 = \sqrt{\rho^2 + ((m+1)a - z + h)^2}$, $r_4 = \sqrt{\rho^2 + (ma - z + h)^2}$.

Hence, combining (17), (18) with (7) provides the explicit expression to the vertical magnetic field in air in the far-region as follows

$$H_{z_0}(\rho,z) = \frac{IdA\,k_0^2\,\rho^2}{4\pi}\left[\begin{array}{c} \sum_{m=0}^{\infty}\left(\frac{1-n}{1+n}\right)^m e^{-2ik_1 md}\left(\frac{1}{r_1^3}\,e^{-ik_0 r_1} - \frac{1}{r_3^3}\,e^{-ik_0 r_3}\right) + \\ \sum_{m=0}^{\infty}\left(\frac{1-n}{1+n}\right)^{m+1} e^{-2ik_1 (m+1)d}\left(\frac{1}{r_4^3}\,e^{-ik_0 r_4} - \frac{1}{r_2^3}\,e^{-ik_0 r_2}\right)\end{array}\right] \tag{19}$$

In a similar manner, the expressions of other field components H_{ρ_0}, E_{φ_0} can be computed.

3.2 Case II (in the air region $0 \le z \le d$):

Here we derive the vertical magnetic field in the overburden slab (upper layer of the earth), where the observation point $P(\rho,\varphi,z)$ is located at any arbitrary distance ($z \neq 0$). The expressions of the field components $E_{\varphi 1}$,$H_{\rho 1}$,H_{z1} can be written as

$$E_{\varphi_1}(\rho,z) = \frac{-i\omega\mu_0 IdA}{8\pi}\int_{-\infty}^{\infty}\left[\begin{array}{c}\frac{\lambda}{u_1}\ e^{u_1(z-h)} + f_1(\lambda)\,e^{-u_1(z-h)} \\ +f_2(\lambda)\,e^{u_1(z-h)}\end{array}\right]\lambda\,H_1^{(2)}(\lambda\rho)\,d\lambda \tag{20a}$$

$$H_{\rho_1}(\rho,z) = \frac{-IdA}{8\pi}\int_{-\infty}^{\infty}\left[\begin{array}{c}\frac{-\lambda}{u_1}\ e^{u_1(z-h)} - f_1(\lambda)\,e^{-u_1(z-h)} \\ +f_2(\lambda)\,e^{u_1(z-h)}\end{array}\right]\lambda\,u_1\,H_1^{(2)}(\lambda\rho)\,d\lambda \tag{20b}$$

$$H_{z_1}(\rho,z) = \frac{IdA}{8\pi}\int_{-\infty}^{\infty}\left[\begin{array}{c}\frac{\lambda}{u_1}\ e^{u_1(z-h)} + f_1(\lambda)\,e^{-u_1(z-h)} \\ +f_2(\lambda)\,e^{u_1(z-h)}\end{array}\right]\lambda^2\,H_0^{(2)}(\lambda\rho)\,d\lambda \tag{20c}$$

where

$$f_1(\lambda) = \frac{\left(\frac{2\lambda}{u_{01}^{+}}\right)e^{-u_{01}^{+}h}}{1 + r_{01}\,r_{12}\,e^{-2u_1 d}}\ ,\quad f_1(\lambda) = \frac{\left(\frac{2\lambda}{u_{01}^{+}}\right)r_{12}\,e^{-u_{01}^{-}h - 2u_1 d} - \left(\frac{\lambda}{u_1}\right)\left(1 + r_{01}r_{12}\,e^{-2u_1 d}\right)}{1 + r_{01}\,r_{12}\,e^{-2u_1 d}} \tag{21}$$

After substituting the value of $f_1(\lambda)$, $f_2(\lambda)$ in (20c), the vertical magnetic field can be written in a simplest form as

$$H_{z_1}(\rho,z) = \frac{IdA}{8\pi}\int_{-\infty}^{\infty}\frac{2\lambda^3}{u_{10}^{\mp}}\left[\frac{e^{-u_0 h - u_1 z} + \frac{u_{12}^{-}}{u_{12}^{\mp}}\,e^{-u_0 h - u_1(2d-z)}}{1 - \frac{u_{10}^{-}\,u_{12}^{-}}{u_{10}^{\mp}\,u_{12}^{\mp}}e^{-2u_1 d}}\right]H_0^{(2)}(\lambda\rho)\,d\lambda \tag{22}$$

Now, with the aid of the complex image theory [16] we will use equation (5) and first formula of (9), then equation (22) will take the following form

$$H_{z_1}(\rho,z) = \frac{IdA}{8\pi}\int_{-\infty}^{\infty}\frac{2\lambda^3}{u_{10}^{\mp}}\left[e^{-u_0h-u_1z}+e^{-u_0h-u_1(2d-z)}\right]\left[\sum_{m=0}^{\infty}\left(\frac{u_{12}^{-}}{u_{12}^{\mp}}e^{-u_0a-2u_1d}\right)^m\right]H_0^{(2)}(\lambda\rho)\,d\lambda \tag{23}$$

Therefore, equation (23) can be written in a simplest form as

$$H_{z_1}(\rho,z) = \frac{IdA}{8\pi}\left[\sum_{m=0}^{\infty}(L(m)+M(m))\right] \tag{24}$$

where

$$L(m) = \int_{-\infty}^{\infty}\frac{2\lambda^3}{u_{10}^{\mp}}\left(\frac{u_{12}^{-}}{u_{12}^{\mp}}\right)^m e^{-u_0(ma+h)-u_1(2md+z)}\,H_0^{(2)}(\lambda\rho)\,d\lambda \tag{25a}$$

$$M(m) = \int_{-\infty}^{\infty}\frac{2\lambda^3}{u_{10}^{\mp}}\left(\frac{u_{12}^{-}}{u_{12}^{\mp}}\right)^{m+1} e^{-u_0(ma+h)-u_1[2(m+1)d+z]}\,H_0^{(2)}(\lambda\rho)\,d\lambda \tag{25b}$$

The integrals $L(m)$ and $M(m)$ evaluated in the same way in (8) and it yields

$$L(m) = 2\rho^2k_0^2\left(\frac{1-n}{1+n}\right)^m e^{-ik_1(2md+z)}\left[\frac{e^{-ik_0r_m}}{r_m^3}-\frac{e^{-ik_0r_{m+1}}}{r_{m+1}^3}\right] \tag{26a}$$

$$M(m) = 2\rho^2k_0^2\left(\frac{1-n}{1+n}\right)^{m+1} e^{-ik_1[2(m+1)d-z]}\left[\frac{e^{-ik_0r_m}}{r_m^3}-\frac{e^{-ik_0r_{m+1}}}{r_{m+1}^3}\right] \tag{26b}$$

where $r_m = \sqrt{\rho^2+(ma+h)^2}$, $r_{m+1} = \sqrt{\rho^2+((m+1)a+h)^2}$.

Then, after substituting from equations (24) - (26) into equation (23), we can find the final value of the vertical magnetic field in the far-region in the overburden slab as

$$H_{z_1}(\rho,z) = \frac{IdA\,k_0^2\rho^2}{4\pi}\left[\begin{array}{c}\sum_{m=0}^{\infty}\left(\frac{1-n}{1+n}\right)^m e^{-2ik_1(2md+z)}\left(\frac{1}{r_m^3}e^{-ik_0r_m}-\frac{1}{r_{m+1}^3}e^{-ik_0r_{m+1}}\right)+\\ \sum_{m=0}^{\infty}\left(\frac{1-n}{1+n}\right)^{m+1} e^{-2ik_1[2(m+1)d-z]}\left(\frac{1}{r_m^3}e^{-ik_0r_m}-\frac{1}{r_{m+1}^3}e^{-ik_0r_{m+1}}\right)\end{array}\right] \tag{27}$$

The expressions of other field components H_{ρ_1} , E_{φ_1} can be computed by the same way.

IV. NUMERICAL RESULTS

The magnitude of the vertical magnetic field both in air and overburden earth at any point has numerically computed for different cases and plotted. In the plotted curves shown in Figs. (2) - (3), the conductivities ratio is taken as $\left[\left(\frac{\sigma_2}{\sigma_1}\right)=100\right]$, $[\sigma_1 = 10^{-3}]$, while the permittivities are $[\varepsilon_1 = 10\,\varepsilon_0]$, $[\varepsilon_2 = 100\,\varepsilon_0]$, and the source height [(h) = 30 m]. Fig. (2) show the variation of the vertical magnetic field in air at [z = 5 m]. While fig. (3) show the vertical magnetic field in the overburden slab at distance [z = 15 m].

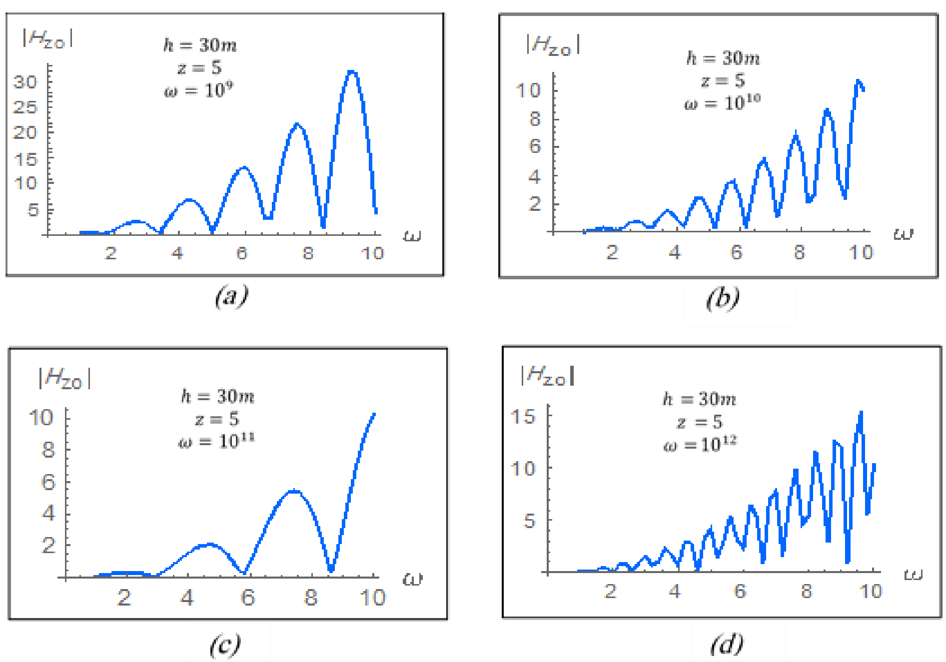

Figure 2. Variation of the vertical magnetic field |Hzo | in frequency - domain in the air region at different values

of ω .

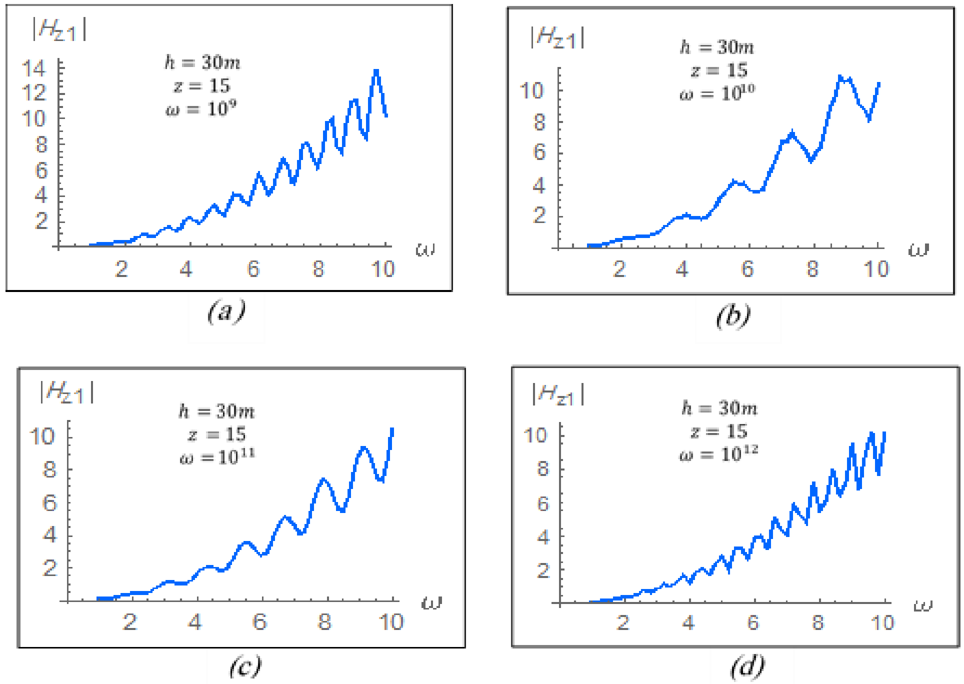

Figure 3: Variation of the vertical magnetic field |Hz1| in frequency - domain in the overburden slab at different values of ω .

V. CONCLUSION

Explicit expressions, valid in a wide frequency range, have been derived for the electromagnetic fields of a small horizontal loop antenna located at certain height above the surface of a homogeneous medium. The result of the analytical integration can be easily performed. The results agree with all the assumptions underlying previously published approximate solutions to this problem and, at the same time, constitute an accurate and quick analytical benchmark used to solve EM boundary value problems.

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