# The Effect of Brownian motion and Thermophoresis on nanofluids stretching for Jaffrey fluid model

Sushma V Jakati[1], Raju B T[2], Achala L Nargund[3], S B Sathyanarayana[4]

[1] *Asistant professor, Research scholar at REVA University Bengaluru, MES College of Arts, Commerce &Science, Bengaluru*

[2]*School of Mathematics, REVA University Bengaluru*

[3]*P.G Department of Mathematics & Research centre in Applied Mathematics, MES College of Arts, Commerce &Science, Bengaluru*

[4]*Asistant professor, Vijaya College Bengaluru*

**Abstract-The effect of Brownian motion and Thermophoresis on nanofluid flow over a stretching sheet is studied using Jaffery fluid model. Reduced ordinary differential equations are solved by semi numerical technique called Homotopy Analysis Method (HAM). The HAM solution is a series solution and is matched with existing numerical solutions. The effect of non dimensional parameters such as Nusslet number Nu, Sherwood number Sh, Prandtle number Pr, Brownian motion parameter $N_b$ Thermophoresis parameter $N_t$ are shown graphically.**

**Keywords Jeffrey fluid model, Nanofluid flow, Stretching sheet, Brownian motion, Thermophoresis, Homotopy Analysis Method.**

## I. INTRODUCTION

The analysis of the flow field in a boundary layer near stretching sheet and moving boundary are an important part of fluid dynamics and heat transfer, which have many applications in engineering and medical industries. Viscosity μ of a fluid plays a very important role in the analysis of liquid behavior and nanofluid motion. Stress is an important force based on which fluid can be divided as Newtonian and non-Newtonian fluids. In Newtonian fluid shearing stress τ is directly proportional to shear rate where as in non-Newtonian fluids shear stress τ is not directly proportional to shear rate $\frac{dv}{dy}$ (deformation rate). Fluids for which the rate of shear at any point is determined by the value of the shear stress at that point, which are time independent, such as Bingham plastic, Pseudo plastic and Dilatants fluid. The time dependent non-Newtonian fluids are Rheopectic and Thixotropic fluids.

Nanofluids are made of nano particles (<100nm) suspended in a base fluids such as water, oil and ethylene glycol. Choi's team [2] discovered that the nano layer acts as a thermal bridge between solid nanoparticle and base fluid. Thus nanofluids provide a significant heat transfer surface between particles and fluids. Also a characteristic feature of nanofluid is thermal conductivity. They are more stable suspension, highly conducting heat transfer fluids Due to the size of the nano particles Brownian diffusion takes places in nanofluids and heat transfer takes places due to Brownian diffusion. Scientist and engineers have introduced that nanofluids are highly conducting heat transfer fluids which enhance the efficiency of large scale heat exchangers used in chemical processing plants, smaller scale heat exchangers used in automotives and also nanofluids for industrial cooling applications result in energy savings, engine cooling /vehicle management. Nanofluids have heat transfer applications as a microelectronic fuel cells, electronic cooling, Domestic refrigerator chiller, solar water heating chiller, heat pipes. Due to their higher thermal conductivity nanofluids are used for liquid cooling computer processors. Combustion of diesel fuel mixed with aluminum nanofluid increase the total combustion heat, decrease the concentration of smoke and nitrous oxide in the exhaust emission from diesel engine hence nanofluid has applications in fuel. Nanofluids are excellent in the processes of soil remediation, lubrication, oil recovery and detergency.

Bio-medical application, which involve nanofluids as drug delivery, hyperthermia, magnetic cell separation. In a cancer therapy some nanofluids are used in cancer imaging, targeted drug delivery and are used to guide the particles up the blood stream to a tumor with magnets. It will allow doctor to deliver high local doses of drugs or radiation without damaging nearby healthy tissues because magnetic nanoparticles are more adhesive to tumor cells.

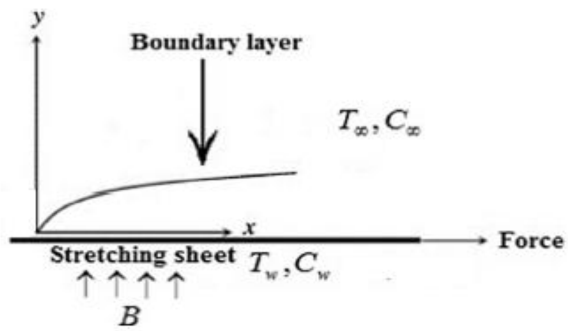
The study of non-Newtonian nanofluid flow and heat transfer over stretching sheets is of considerable interest because of biological, engineering and industrial applications. Crane [3] investigated the concept of flow caused by the stretching sheet. Gupta and Gupta, Datta et al, Chan and Char extended the research work of Crane. Sakiadis [4] presented the concept of boundary layer flow.

Nadeem et al. [6] discussed the effect of thermal radiation on the boundary layer flow of a Jeffery fluid over an exponentially stretching surface. Mohammed et al. [7] investigated the steady two dimensional Magneto Hydro Dynamic (MHD) free convective boundary layer flow of a Newtonian nanofluid over a flat solid vertical plate with Newtonian heating boundary condition in a quiescent fluid. It is found that the rate of heat and mass transfer increases as Newtonian heating parameter increases. Hyder et al. [8] analyzed the effect of oxides nanoparticle materials on the pressure loss and heat transfer of nanofluids in circular pipes. It is found that pressure loss increases with the particle volume concentration and enhancement of heat transfer is better with increase of Renold's number of the flow. Rashidi [9] studied theoretically the effects of Brownian motion and Thermophoresis on natural convective boundary layer flow of a nanofluid over a vertical surface. Ternik and Rudolf [10] examined the heat transfer enhancement for natural convection flow of water based nanofluids in a square enclosure. In comparison to convectional fluids they found that heat transfer enhancement is possible using nanofluids. Sohail Nadeem and Changhoon Lee [11] investigated the steady boundary layer flow of nanofluid over an exponentially stretching surface analytically by applying HAM. Siddique et al. [12] have applied the Adomian decomposition method to solve nonlinear differential equations arises in non-Newtonian fluids. Qasim. [14] Studied the combined effect of heat and mass transfer in Jeffery fluid over stretching sheet in presence of heat source. Anabazhagan et al. [16] have analyzed that the improving the thermal conductivity is the idea to improve heat transfer in conventional nanofluids. Enhancement of thermal conductivity of conventional nanofluids is by the suspension of solid metal particles into the base fluid. Stanford Shateyi [17, 24] studied the effect of Thermophorosis and chemical reaction to MHD flow of a Maxwell fluid past a vertical stretching sheet. Thermophorosis causes small particles to deposit on the cold surface which has several applications in aerosol technology, silicon thin films and radioactive particle deposition in nuclear reactor. He also investigated the MHD flow and heat transfer by spectral relaxation method. Sohail Nadeem et al. [18] studied numerically that the boundary layer flow and heat transfer of Oldroyd-B nanofluid towards a stretching sheet. Krishnendu Bhattacharyya [15] analyzed the steady boundary layer stagnation-point flow of Casson fluid and heat transfer towards a stretching/shrinking sheet. Nadeem S et al. [20] studied the nanoparticle concentration for the non-Newtonian Jeffrey fluid model considered with the process of peristaltic waves in a three-dimensional rectangular channel. Khairy Zaimi el al. [26] investigated the steady two-dimensional flow and heat transfer over a stretching/shrinking sheet in a nanofluid using Buongiorno's nanofluid model introduced by Buongiorno in 2006. Srikant et al. [27] studied the mass transfer with MHD nanofluids due to Brownian motion of particles During the mass transfer they analyzed the effect of chemical reaction on mass transfer. Kalidas Das et al. [28] investigated numerically the influence of melting heat transfer and thermal radiation on MHD Jeffery fluid over stretching with surface slip. Nadeem et al. [23] studied the nanoparticle concentration for the non-Newtonian Jeffrey fluid model considered with the process of peristaltic waves in a three-dimensional rectangular channel. Zeeshan and Majeed [29] investigated the effect of magnetic dipole and heat transfer in the analysis of Jeffery fluid flow over stretching sheet with suction/injection. Hayat et al. [30] discussed the MHD flow of Jeffery liquid to a nonlinear radially stretched sheet. They studied that heat transfer is due to effect of Newtonian heating and Joule heating. Ghafouri et al. [31] studied numerically the effect of variable thermal conductivity models on the combined convection heat transfer in a square enclosure filled with water-alumina nanofluid.

Nanofluid with Jaffrey model is studied numerically by many methods. We are giving analytical solution to two dimensional steady non-Newtonian incompressible nanofluid for Jaffrey model. Paper consist of sections, section1 is introduction, section 2 Geometry and Basic equations, section 3 Methodology, section 4 is Result, section 5 contains Discussion, section 6 Graph and section 7 References.

## II. BASIC EQUATIONS

In this paper we have studied the steady two dimensional flow of an incompressible nanofluid over a stretching sheet with nano particles. x-axis is taken along with the stretching sheet in the direction of motion by assuming that the sheet is stretched with the linear velocity $u_m(x) = ax$, where $a>0$. The flow and the heat transfer characteristic of Jaffery fluid along with nano particles are governed by the following equations.




Physical model and coordinate system.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{v}{1+\lambda}\left[\frac{\partial^2 u}{\partial y^2} + \lambda 1\left(u\frac{\partial^3 u}{\partial x \partial y^2} - \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x \partial y} + v\frac{\partial^3 u}{\partial y^3}\right)\right], \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \tau\left[D_B\left(\frac{\partial C}{\partial x}\frac{\partial T}{\partial x} + \frac{\partial C}{\partial y}\frac{\partial T}{\partial y}\right) + \left(\frac{D_T}{T_\infty}\right)\left\{\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2\right\}\right], \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right) + \left(\frac{D_T}{T_\infty}\right)\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right), \tag{4}$$

u is the velocity in the direction of x, v is the velocity in the direction of y, $\rho_f$ is the density of the base fluid ,σ is the electrical conductivity, λ is the ratio of relaxation to retardation times parameter, $\lambda_1$ is the retardation time parameter, α the thermal diffusivity, T is the fluid temperature, C Nanoparticles fraction, $T_W$ temperature of fluid, $C_W$ Temperature of nanoparticles fraction at wall, $D_B$ the Brownian diffusion coefficient, $D_T$ the Thermophoretic diffusion coefficient, $\rho_p$ is the density of the particles, β Deborah number, Pr Prandtl number, $N_b$ Brownian motion parameter, $N_t$ Thermophoresis parameter, Le be the Lewis number, Nu Local Nusselt number, Sh be the Local Sherwood number, $q_w$ is the heat flux, $q_m$ is the mass flux, $Re_x$ Local Renolds number, $u_w(x)$ stretching velocity, $(\rho c)_p$ is the effective heat capacity of nano particle material, $(\rho c)_f$ is the heat capacity of the base fluid, τ the ratio of heat capacity of nano particle to the heat capacity of the base fluid, γ is the Kinematic viscosity of the fluid.

$$Pr = \frac{v}{\alpha}\ , Le = \frac{\alpha}{D_B}\ ,\ \beta = \lambda_1 a\ , \tau = \frac{(\rho c)_p}{(\rho c)_f}, \tag{5}$$

$$N_b = \frac{(\rho c)_p D_B\,(\varphi_w - \varphi_\infty)}{v\,(\rho c)_f},\ \ N_t = \frac{(\rho c)_p D_T\,(T_w - T_\infty)}{v(\rho c)_f}, \tag{6}$$

$$Nu = \frac{x q_w}{\alpha(T_w - T_\infty)}, \tag{7}$$

$$Sh = \frac{X q_m}{D_B\,(C_w - C_\infty)}, \tag{8}$$

$$q_w = -\alpha\left(\frac{\partial T}{\partial y}\right)_{y=0},\ q_m = -D_B\left(\frac{\partial G}{\partial y}\right)_{y=0.} \tag{9}$$

The associated boundary conditions are

$$As\ y \to \infty, u = 0, v = 0, u_y = 0, T \to T_\infty, C \to C_\infty \tag{10}$$

$$At\ x - axis, u = u_w\,(x) = ax, v = 0, T = T_w, C = C_w \tag{11}$$

Introducing the following similarity transformations

$$\Psi = (av)^{\frac{1}{2}} x f(\eta) \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{12}$$

$$\varphi(\boldsymbol{\eta}) = \frac{C - C_\infty}{C_w - C_\infty} \quad \eta = \sqrt{\frac{a}{\nu}}\, y \tag{13}$$

where the stream function $\Psi$ is defined as $u = \frac{\partial \Psi}{\partial y}$ *and* $v = -\frac{\partial \Psi}{\partial x}$

Using similarity transformation and associated boundary conditions, the continuity equation is identically satisfied, momentum equation and energy equation reduces to ODE

The governing coupled non linear equations for this problem with the boundary condition are

$$f''' + \beta(f''^2 - ff'''') + (1+\lambda)\left(ff'' - f'^2\right) = 0, \tag{14}$$

$$\theta'' + \Pr\left[f\theta' + N_b\varphi'\theta' + N_t\,\theta'^2\right] = 0, \tag{15}$$

$$\varphi'' + \mathrm{LePr}(f\varphi') + \frac{N_t}{N_b}\theta'' = 0, \tag{16}$$

$$f(0) = 0, f'(0) = 1, f'(\infty) = 0, f''(\infty) = 0, \tag{17}$$

$$\theta(0) = 1, \theta(\infty) = 0, \tag{18}$$

$$\varphi(0) = 1, \varphi(\infty) = 0. \tag{19}$$

## III. HOMOTOPY ANALYSIS METHOD FOR NONLINEAR BOUNDARY VALUE PROBLEM

Achala and Sathyanarayana [13] discussed the nonlinear boundary value problems by homotopy analysis method.The governing coupled non linear equations for this problem is

$$N[f(\eta)] = f''' + \beta(f''^2 - ff'''') + (1+\lambda)\left(ff'' - f'^2\right), \tag{20}$$

$$N[\theta(\eta)] = \theta'' + \Pr\left[f\theta' + N_b\varphi'\theta' + N_t\,\theta'^2\right], \tag{21}$$

$$N[\varphi(\eta)] = \varphi'' + \mathrm{LePr}(f\varphi') + \frac{N_t}{N_b}\theta''. \tag{22}$$

We can choose an auxiliary linear operator for the equation (20), (21), (22) respectively as

$$L_f = \frac{\partial^3}{\partial\eta^3} + \frac{\partial^2}{\partial\eta^2} \quad L_\theta = \frac{\partial^2}{\partial\eta^2} + \frac{\partial}{\partial\eta} \quad L_\varphi = \frac{\partial^2}{\partial\eta^2} + \frac{\partial}{\partial\eta}. \tag{23}$$

By using boundary conditions (17), (18), (19) and applying linear operators (23) for f, θ, φ we get the initial approximation respectively as $f_0(\eta)$, $\theta_0(\eta)$, $\varphi_0(\eta)$ given by

$$f_0(\eta) = \phi_0(\eta) = 1 - e^{-\eta}, \tag{24}$$

$$\theta_0(\eta) = \Psi_0(\eta) = e^{-\eta}, \tag{25}$$

$$\varphi_0(\eta) = \xi_0(\eta) = e^{-\eta}, \tag{26}$$

When p=0 and p=1 we have

$$F(\eta, 0) = f_0(\eta) \quad F(\eta, 1) = f(\eta) \tag{27}$$

$$G(\eta, 0) = \theta_0(\eta) \quad G(\eta, 1) = \theta(\eta). \tag{28}$$

$$E(\eta, 0) = \varphi_0(\eta) \quad E(\eta, 1) = \varphi(\eta). \tag{29}$$

Thus as p increases from 0 to 1, the solution $f_0(\eta)$ , $\theta_0(\eta)$, $\varphi_0(\eta)$ varies from the initial guess to the exact solution $f(\eta)$ , $\theta(\eta)$, $\varphi(\eta)$. Homotopy equations for (20), (21), (22) are constructed as below

$$(1-p)L[F(\eta,p) - f_0(\eta)] = hp[\frac{\partial^3 F}{\partial\eta^3} + \beta\left\{\left(\frac{\partial^2 F}{\partial\eta^2}\right)^2 - F\frac{\partial^4 F}{\partial\eta^4}\right\} + (1+\lambda)\left\{F\frac{\partial^2 F}{\partial\eta^2} - \left(\frac{\partial F}{\partial\eta}\right)^2\right\}] \tag{30}$$

$$(1-p)L[G(\eta,p) - \theta_0(\eta)] = hp[\frac{\partial^2 G}{\partial\eta^2} + \Pr\left\{F\frac{\partial G}{\partial\eta} + N_b\frac{\partial G}{\partial\eta}\frac{\partial E}{\partial\eta} + N_t\left(\frac{\partial G}{\partial\eta}\right)^2\right\}] \tag{31}$$

$$(1-p)L\left[E(\eta,p) - \varphi_0(\eta)\right] = hp\left\{\frac{\partial^2 E}{\partial\eta^2} + \mathrm{Le}\,\Pr\,F\frac{\partial E}{\partial\eta} + \frac{Nt}{Nb}\frac{\partial^2 G}{\partial\eta^2}\right\} \tag{32}$$

Obviously conditions are

$$F(0,p) = 0, F_{\eta}(0,p) = 1,\ F_{\eta}(\infty,p) = 0,\ F_{\eta\eta}(\infty,p) = 0, \tag{33}$$

$$G(0,p) = 1, G(\infty,p) = 0, \tag{34}$$

$$E(0,p) = 1, E(\infty,p) = 0. \tag{35}$$

Maclaurin's series for $F(\eta,p)$, $G(\eta,p)$ and $E(\eta,p)$ are

$$F(\eta,p) = F(\eta,0) + \sum_{k=1}^{\infty} \frac{p^k}{k!} \frac{\partial^k F(\eta,p)}{\partial p^k}, \tag{36}$$

$$G(\eta,p) = G(\eta,0) + \sum_{k=1}^{\infty} \frac{p^k}{k!} \frac{\partial^k G(\eta,p)}{\partial p^k}, \tag{37}$$

$$E(\eta,p) = E(\eta,0) + \sum_{k=1}^{\infty} \frac{p^k}{k!} \frac{\partial^k E(\eta,p)}{\partial p^k}, \tag{38}$$

now by defining

$$\Phi_0(\eta) = F(\eta,0) = f_0(\eta), \tag{39}$$

$$\Psi_0(\eta) = G(\eta,0) = \theta_0(\eta), \tag{40}$$

$$\xi_0(\eta) = E(\eta,0) = \varphi_0(\eta),. \tag{41}$$

We get

$$F(\eta,p) = \phi_0(\eta) + \sum_{k=1}^{\infty} \phi_k(\eta) p^k, \tag{42}$$

$$G(\eta,p) = \Psi_0(\eta) + \sum_{k=1}^{\infty} \Psi_k(\eta) p^k, \tag{43}$$

$$E(\eta,p) = \xi_0(\eta) + \sum_{k=1}^{\infty} \xi_k(\eta) p^k. \tag{44}$$

The convergence region of the above series depends upon the auxiliary linear operator L, and the non-zero auxiliary parameter h which are to be selected such that solution converges at p = 1.

$$f(\eta) = \phi_0(\eta) + \sum_{k=1}^{\infty} \phi_k(\eta) \tag{45}$$

$$\theta(\eta) = \Psi_0(\eta) + \sum_{k=1}^{\infty} \Psi_k(\eta) \tag{46}$$

$$\varphi(\eta) = \xi_0(\eta) + \sum_{k=1}^{\infty} \xi_k(\eta). \tag{47}$$

Differentiating equation (30), (31) and (32) m times about the embedding parameter p, using Leibnitz theorem, setting p = 0 and dividing by m! we get

$$L_f[\phi_m - \chi_m \phi_{m-1}] = hR_m(\eta), \tag{48}$$

$$L_\theta[\Psi_m - \chi_m \Psi_{m-1}] = hS_m(\eta), \tag{49}$$

$$L_\varphi[\xi_m - \chi_m \xi_{m-1}] = hT_m(\eta). \tag{50}$$

where $\chi_m = \begin{cases} 0 & \text{when } m \leq 1 \\ 1 & \text{when } m > 1 \end{cases}$ and (51)

$$R_m(\eta) = \phi_{m-1}'''(\eta) + \beta \sum_{k=0}^{m-1} \phi_{m-1-k}''(\eta)\phi_k''(\eta) - \beta \sum_{k=0}^{m-1} \phi_{m-1-k}(\eta)\phi_k^{IV}(\eta) + (1 + \lambda) \sum_{k=0}^{m-1} \phi_{m-1-k}(\eta)\phi_k''(\eta) - (1+\lambda) \sum_{k=0}^{m-1} \phi_{m-1-k}'(\eta)\phi_k'(\eta) \tag{52}$$

$$S_m(\eta) = \Psi_{m-1}''(\eta) + Pr \sum_{k=0}^{m-1} \phi_{m-1-k}(\eta)\Psi_k'(\eta) + N_b\, Pr \sum_{k=0}^{m-1} \Psi_{m-1-k}'\xi_k'(\eta) + Pr N_t \sum_{k=0}^{m-1} \Psi_{m-1-k}'(\eta)\Psi_k'(\eta) \tag{53}$$

$$T_m(\eta) = \xi_{m-1}''(\eta) + Le\, Pr \sum_{k=0}^{m-1} \phi_{m-1-k}(\eta)\xi_k'(\eta) + \frac{N_t}{N_b} \Psi_{m-1}''(\eta) \tag{54}$$

with boundary conditions

$$\phi_m(0) = 0\ \phi_m'(0) = 0\ \phi_m'(\infty) = 0\ \phi_m''(\infty) = 0, \tag{55}$$

$$\Psi_m(0) = \Psi_m(\infty) = 0, \tag{56}$$

$$\xi_m(0) = 1, \xi_m(\infty) = 0. \tag{57}$$

We solve these non-linear equations given by (48), (49) and (50) for $\phi_m$, $\Psi_m$, $\xi_m$ by MATHEMATICA. Using these coefficients in (45), (46) and (47) we get the solution of the given equations. Data of above solutions are analyzed through graphs for different characteristic parameters.

## IV. RESULT AND DISCUSSION

The solutions for (52)-(54) are obtained with the help of Mathematica and discussed through graphs. Convergence of the series solution depends on the non-zero auxiliary parameter h. in fig1 velocity profile $f'(\eta)$ increases with the increase in the value of Deborah number $\beta$ where as temperature profile $\theta(\eta)$, and nanoparticle fraction profile $\varphi(\eta)$ decreases for the increase in the value of $\beta$. In fig2 velocity decreases with an increase in the value of the parameter $\lambda$ whereas temperature and nanoparticle fraction increases for increase in the value of $\lambda$. In fig3 temperature increases as the value of Brownian motion parameter $N_b$ increases. This shows that the thermal conductivity of a nanofluid is due to Brownian motion. But higher the values of $N_b$, nanoparticle fraction profile reduces. In fig 4 both temperature profile and nanoparticle fraction profile increases with the increase in the value of Thermophorosis parameter $N_t$. mean while from both fig 3 & 4 we can conclude there is an enhancement in temperature for the large values of $N_b$ and $N_t$ while different behaviour is observed for nanoparticle fraction with the increase in the values of $N_b$ and $N_t$. in fig5 we observed the effect of Pr on $\theta(\eta)$ and $\varphi(\eta)$. Pr is the ratio of viscous diffusion rate to the thermal diffusion rate. Higher the Pr lowers the thermal diffusivity. Higher the Pr, temperature profile and nanoparticle fraction decreases. In fig6 we observed that $\theta(\eta)$ and $\varphi(\eta)$ behave opposite way with the increase in value of Lewis number Le.

## V. CONCLUSIONS

In this study we observed that the effect of Brownian motion and Thermophorosis on nanofluids for Jeffery fluid over a stretching sheet. Semi analytical method HAM works well for non-linear differential equations. We have shown that HAM solution exactly matching with numerical result obtained by Nadeem [20] by 4$^{th}$ order Runge Kutta Fehelberg method with a shooting technique.

## VI. GRAPHS

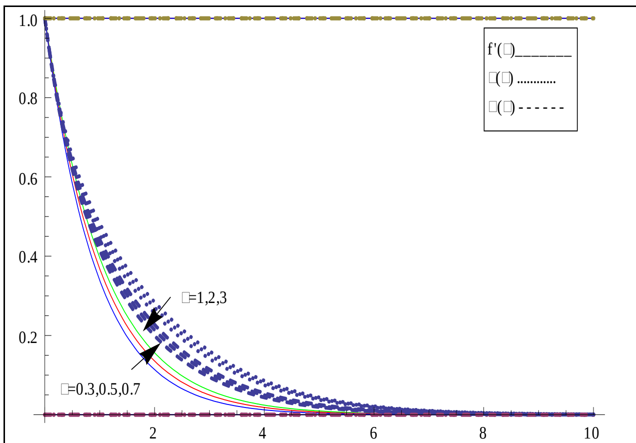



Fig1.Velocity, Temperature distribution, nano particle fraction for different values of β

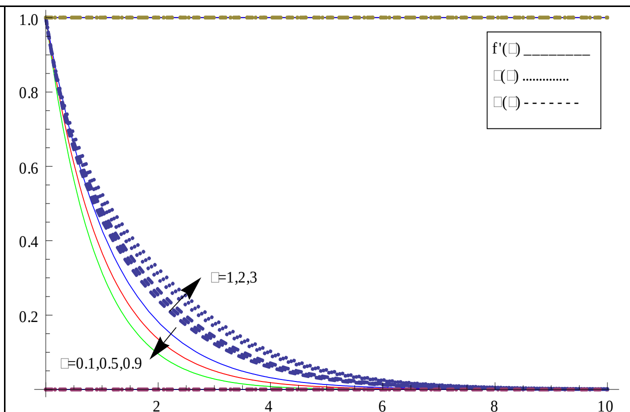



Fig2.Velocity, Temperature distribution, nano particle fraction for different values of λ

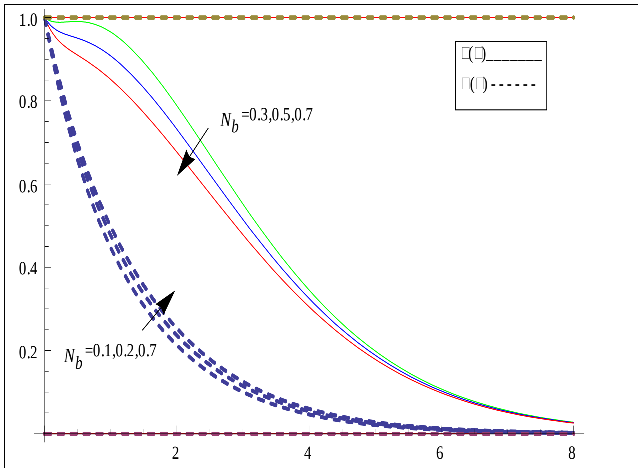



Fig3. Temperature distribution, nano particle fraction for different values of $N_b$

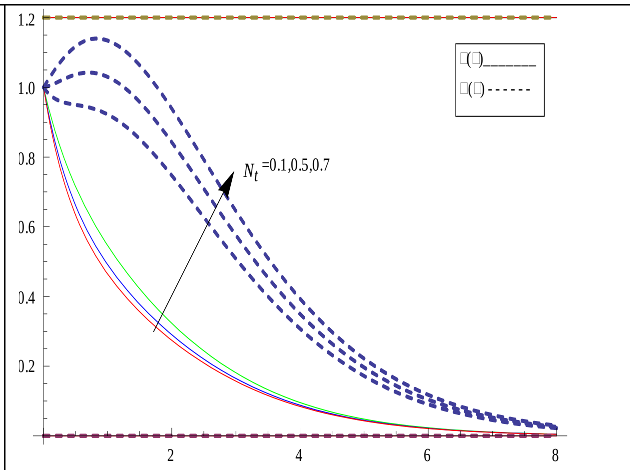



Fig4. Temperature distribution, nano particle fraction for different values of $N_t$

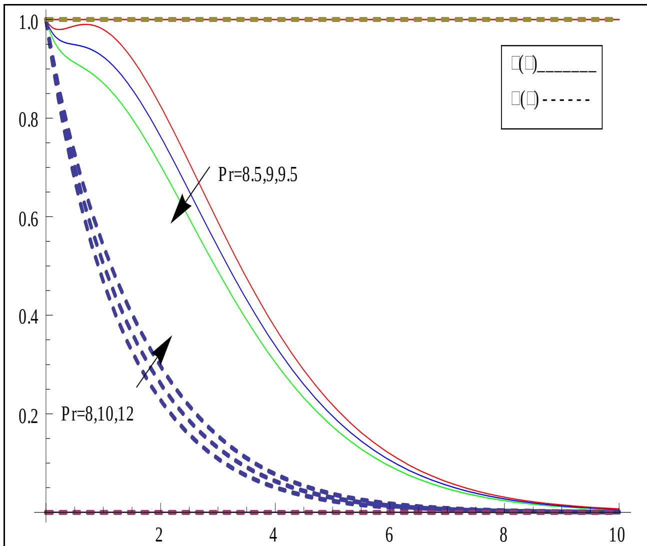



Fig5. Temperature distribution, nano particle fraction for different values of Pr

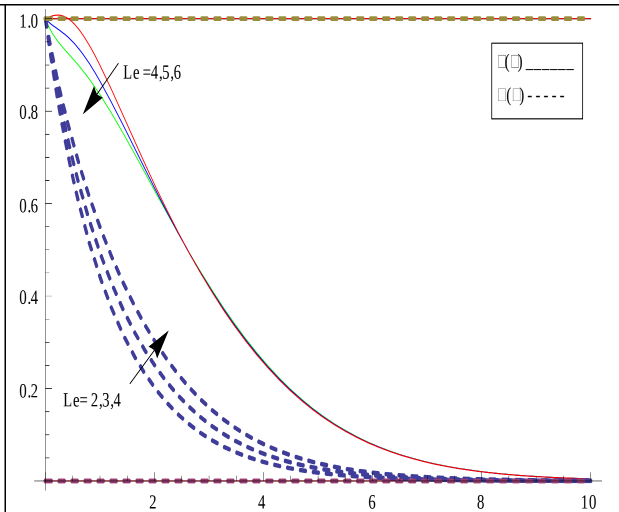



Fig6. Temperature distribution, nano particle fraction for different values of Le

Nomenclature:

u velocity in the direction of x
v velocity in the direction of y
$\rho_f$ Density of the base fluid
σ Electrical conductivity
λ Ratio of relaxation to retardation times parameter
$\lambda_1$ Retardation time parameter
α Thermal diffusivity
T Fluid temperature
C Nanoparticles fraction
$T_W$ Temperature of fluid
$C_W$ Temperature of nanoparticles fraction at wall
$D_B$ Brownian diffusion coefficient

$D_T$ Thermophoretic diffusion coefficient
$\rho_p$ Density of the particles
$\beta$ Deborah number
Pr Prandtl number
$N_b$ Brownian motion parameter
$N_t$ Thermophoresis parameter
Le Lewis number
Nu Local Nusselt number
Sh Local Sherwood number
$q_w$ Heat flux
$q_m$ Mass flux
$Re_x$ Local Renolds number
$u_w(x)$ stretching velocity
$(\rho c)_p$ Effective heat capacity of nano particle material
$(\rho c)_f$ Heat capacity of the base fluid
$\tau$ Ratio of heat capacity of nano particle to the heat capacity of the base fluid
$\gamma$ Kinematic viscosity of the fluid

## VII. REFERENCES


[1] S. J. Liao (1992) "The proposed homotopy analysis techniques for the solution of nonlinear problems", Ph.D. Dissertation, Shanghai Jiao Tong University, Shanghai, China
[2] Choi SUS (1995) "Enhancing thermal conductivity of fluids with nanoparticles, International Mechanical Engineering Congress and Exposition", San Francisco, USA, ASME, FED 231/MD, vol66, 99–105
[3] Crane L (1970) Flow past a stretching plate. Z Angew Math Phys vol21 645–647
[4] Sakiadis BC (1961) "Boundary-layer behavior on continuous solid surface: I Boundary-layer equations for two-dimensional and axisymmetric flow", J Am Inst Chem Eng vol7 26–28
[5] S. J. Liao (2004), "Beyond Perturbation: Introduction to the Homotopy Analysis Method", vol2 of CRC Series: Modern Mechanics and Mathematics, Chapman and Hall/CRC, Boca Raton, Fla, USA
[6] Achala. L. N and Sathyanarayana. S. B, (2011) "Fluid over nonlinearly stretching sheet with magnetic filed by homotopy analysis method ", Jl of Applied Mathematics and fluid mechanics , vol.3, No. 1,15-22
[7] Mohammed. J. Uddin et al. (2012) "MHD free convective boundary layer flow of a nanofluid past a flat vertical plate with Newtonian heating boundary condition", PLOS ONE 7(11) E49499 jol pone.0049499
[8] Hyder. H. Balla et al. (2012) "Effect of oxides nanoparticle materials on the pressure loss and heat transfer of nanofluids in circular pipes", jl of Applied sciences, vol12, 1306-1401
[9] Rashidi. M. M (2012) "DTM-Pade Modelling of natural convective boundary layer flow of a nanofluid past a vertical surface", Int.J of Thermal and environmental engineering, vol 4, No 1 13-24.
[10] Ternik and Rudolf (2012) "Heat transfer enhancement for natural convection flow of water based nanofluids in a square enclosure", Int J siul model vol11, 29-39.
[11] Sohail Nadeem and Changhoon Lee (2012) "Boundary layer flow of nanofluid over an exponentially stretching surface", Nanoscale research letters, Springer Open journal.
[12] Siddique A. M. et al. (2012) "Adomian decomposition method applied to study nonlinear equations arising in non-Newtonian flows", Applied Mathematical sciences, vol6, no 98 4889-4909.
[13] Achala. L. N and Sathyanarayana. S. B, (2013) "Nonlinear boundary value problems by homotopy analysis method ", Jl Applied Mathematics, vol5, 27-46
[14] M. Qasim (2013) "Heat and mass transfer in a Jeffrey fluid over a stretching sheet with heat source/sink", Alexandria engineering journal, vol52, 571-575
[15] Krishnendu Bhattacharya (2013), "MHD Stagnation-Point Flow of Casson Fluid and Heat Transfer over a Stretching Sheet with Thermal Radiation" ,Journal of thermodynamics, vol2013 article ID 169674
[16] Anabazhagan. V. N et al. (2013) "Analysis heat transfer co-efficient of nanofluids", International journal of engineering research and technology (IJERT), vol2, issue 6, 1839-1850
[17] Stanford Shateyi (2013) "A new numerical approach to MHD flow of a Maxwell fluid past a vertical stretching sheet in presence of Thermophorosis and chemical reaction", Springer open logo. 1687-2770-2013:196
[18] Sohail Nadeem et al. (2013) "Numerical Study of boundary layer flow and heat transfer of Oldroyd-B nanofluid towards a stretching sheet", PLOS ONE Jl Pone: 0069811
[19] Krishnendu Bhattacharyya (2013) "Boundary layer stagnation-point flow of Casson fluid and heat transfer towards a stretching/shrinking sheet", Frontiers in heat and mass transfer (FHMT), vol4, 023003
[20] Nadeem. S et.al (2014),"Mathematical model for the peristaltic flow of Jeffert fluid with nanoparticles phenomenon through a rectangular duct", Applied Nanoscience, vol4, 613-624.

[21] Achalal. L. N and Sathyanarayana. S. B, (2014) "Approximate Analytical solution of compressible boundary layer flow over a flat plate with pressure gradient", Theoretical Applied Mathematics, vol5, 15-31,

[22] Achalal. L. N and Sathyanarayana. S. B, ( 2014) "Approximate Analytical solution of Magneto Hydro Dynamics compressible boundary layer flow with pleasure gradient and suction/injection", Candian journal of physics, vol5, 15-31

[23] S. Nadeem et al, (2014) "Numerical solution of non-Newtonian nano fluid over stretching sheet", Appl Nanosci, vol 4, 625-631

[24] Stanford Shateyi (2014) "On spectral relaxation method for an MHD flow and heat transfer of a Maxwell fluid", ISBN 978 -1-61804-220-0

[25] Khairy Zaimi el al. (2014) "Flow a permeable stretching/shrinking sheet in a nanofluid using two-phase model", PLoS ONE, journal pone 0111743.

[26] Srikant G V P N et al. (2014) "Heat and mass transfer with MHD nanofluid with chemical reaction effects", Internationl journal of mechanical and production engineering, vol2, issue 3.

[27] Kalidas Das et al. (2015) "Radiative flow of MHD Jeffery fluid over stretching sheet with surface slip and melting heat transfer", Alexandria Engineering journal, 1110-0168, Production and Hosting by Elsevier B. V

[28] A. Zeeshan and A. Majeed (2016) "Heat transfer analysis of Jeffery fluid flow over a stretching sheet with suction/injuction and magnetic dipole effect", Alexandria engineering journal, vol55, 2171-2181

[29] T. Hayat et al, (2016), "MHD flow of Jeffrey liquid due to a nonlinear radially stretched sheet in presence of Newtonian heating", Results in Physics, vol6, 817-823, Published by Elsevier B.V.

[30] Ghafouri et al. (2017) "Effect of variable thermal conductivity models on the combined convection heat transfer in a square enclosure filled with water-alumina nanofluid", journal of applied mechanics and technical physics, vol58, no 1.103-115